AlterMundus

Alain Matthes

Collection AlterMundus

tkz-2d.sty

tkz-2d.sty v3.0 04 10 2007

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*The package **tkz-2d.sty** is a set of convenient macros for drawing in a plane (fundamental two-dimensional object) with a Cartesian coordinate system. **tkz-2d.sty** is built on top of PGF and its associated front-end, TikZ and is a (La)TeX-friendly drawing package. The aim is to provide a high-level user interface to build graphics relatively simply. This document provides a collection of graduated examples around the definition of each macro.*

doc-tkz-2d v3.0 04 10 2007

- Firstly, I would like to thank **Till Tantau** for the beautiful LATEX package, namely TikZ.
- I am grateful to **Michel Bovani** for providing the `fourier` font.
- I received much valuable advice from **Jean-Côme Charpentier** and **Josselin Noirel**.

Contents

A list of all macros in the order in which they appear in this documentation.

I. Installation.

You could simply create a folder (directory) prof which path is : texmf/tex/latex/prof. texmf is generally the personnal folder, here ways of this folder on my two computers:

- with OS X **/Users/ego/Library/texmf** ;
- with Ubuntu **/home/ego/texmf**.

If you choose a custom location for your files, I suppose that you know why! The installation that I propose, is valid only for one user.

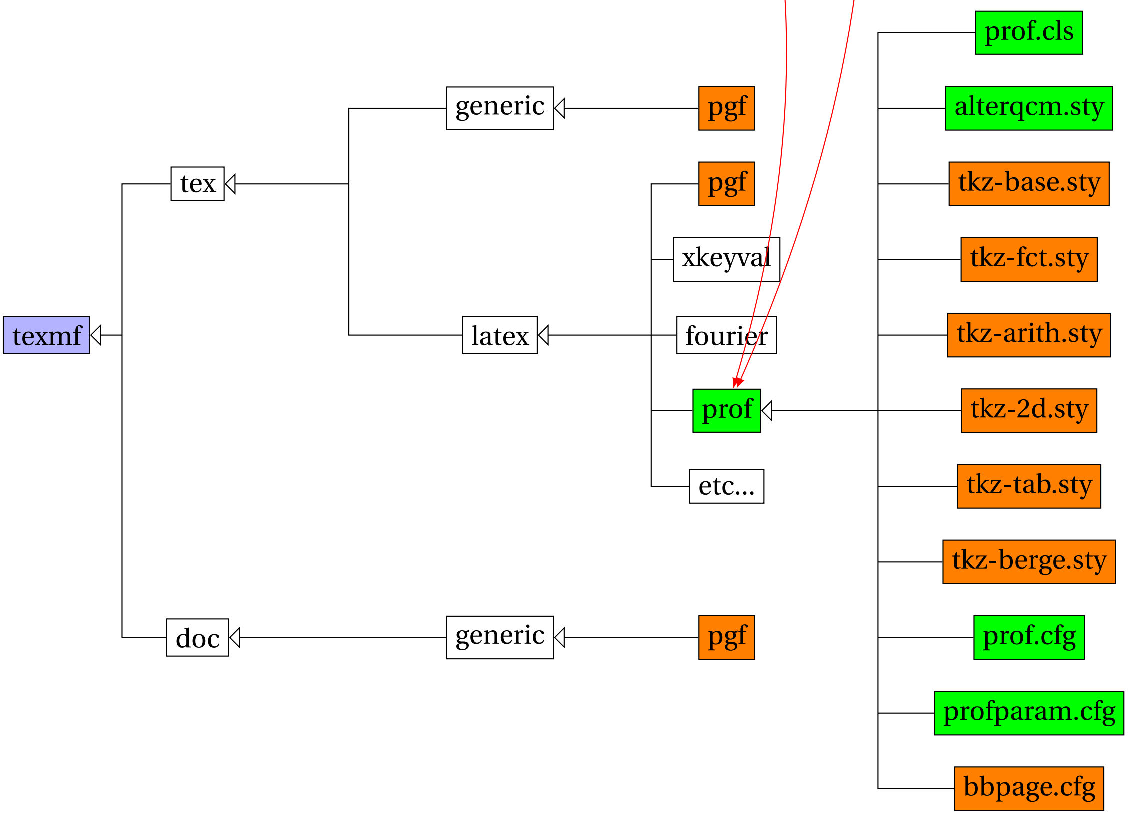
1/ Store the files tkz-2d.sty, tkz-base.sty and tkz-arith.sty in the folder prof.

2/ Open a terminal, then type `sudo texhash`

3/ Check that xkeyval, ifthen, fp et tikz 1.18 are installed because they are obligatory.
⚠ tikz 1.18 is now sufficient but it is preferable to update some files of the math folder. The path of this folder is **texmf/tex/generic/pgf/math**. You can download it here:
`http://pgf.cvs.sourceforge.net/pgf/pgf/generic/pgf/math/`

An other important file is "pgffor.code.tex" here : `http://pgf.cvs.sourceforge.net/pgf/pgf/generic/pgf/utili`

My folder texmf is structured as in the diagram below:



II . Tests

Firstly, you need to test if tkz-base is installed. To determine if this package is correctly installed you need to try the code of the file named "test1.tex".
This package needs the packages fp.sty , ifthen.sty and xkeyval(2.5) (or better).
If you want only to work with 2d.sty, you must know that 2d.sty calls tkz-base. 2d.sty is independant of fp.sty but for the moment, tkz-base uses fp.sty.

A common mistake is to put space between arguments, so try to avoid this.

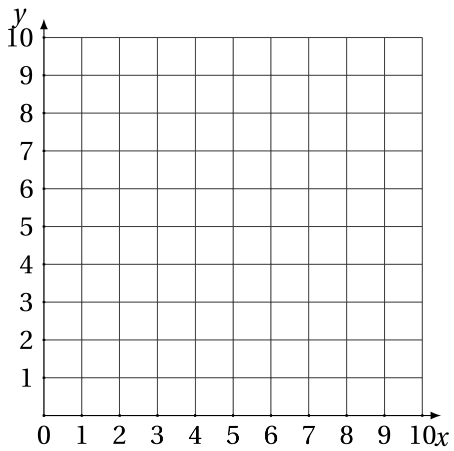
The file pgfmathparser.code.tex of PGF 1.18 must be replace by that which is available in CVS.
You can find this file here :
http://pgf.cvs.sourceforge.net/*checkout*/pgf/pgf/generic/pgf/math/pgfmathparser.code.tex
A better solution is to take all the files of the directory generic/pgf/math

Partie A Test of tkz-base

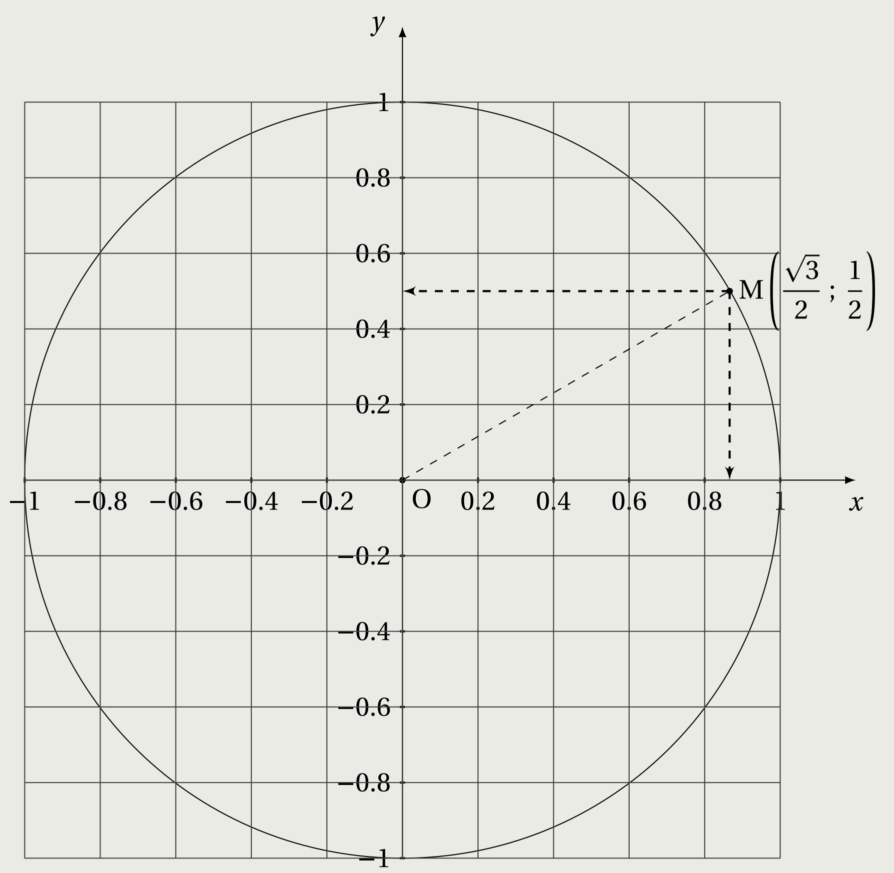
You can type the next code or use the file test1.tex

```
\documentclass{article}
\usepackage{amsmath}
\usepackage{tikz,tkz-base}
\usetikzlibrary{arrows,plotmarks}%
\usepackage[english]{babel}
% pour les français \usepackage[frenchb]{babel}
\usepackage[np,autolanguage]{numprint}
\begin{document}
  \begin{tikzpicture}[scale=.5]
     \tkzInit
     \tkzGrid
     \tkzX[orig]
     \tkzY
  \end{tikzpicture}
\end{document}
```

The result must be :



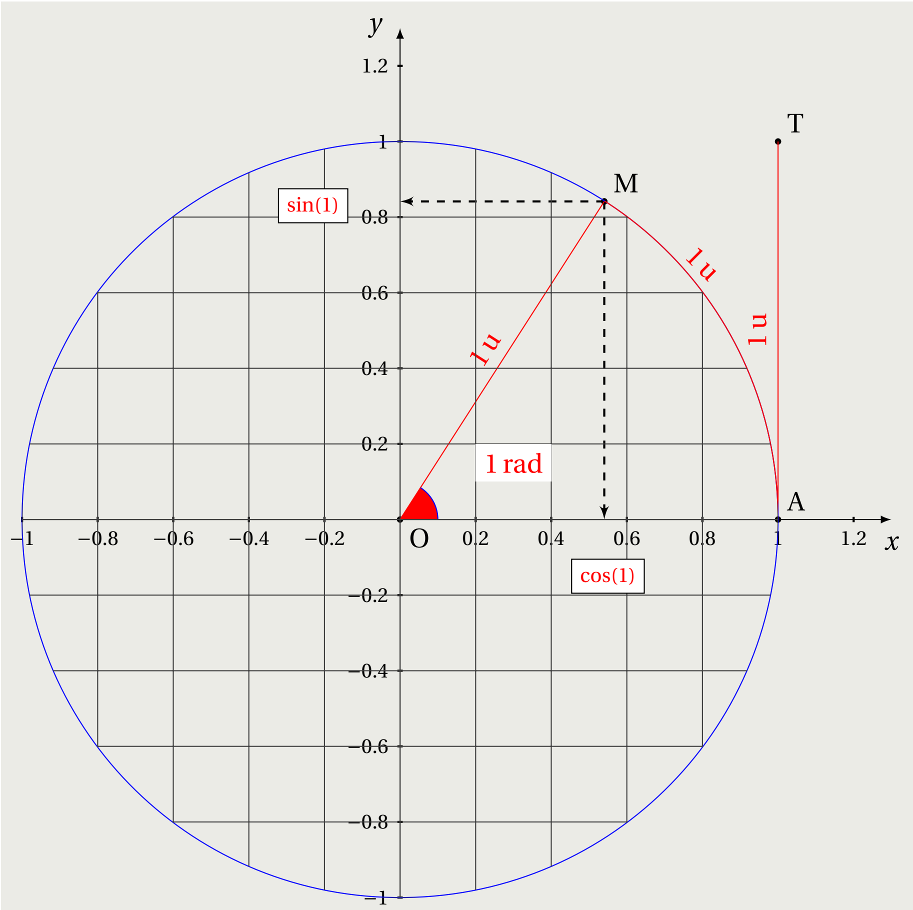
An other test with Tikz 1.18 and the pgfmath library (test2.tex)




```
\begin{tikzpicture}%
   \tkzInit[xmin=-1,xmax=1.1,xstep=.2,ymin=-1,ymax=1.1,ystep=.2]
   \tkzX
   \tkzY
   \tkzPoint(0,0){O}
   \draw (O) circle (5cm);
   \tkzGrid(-1,-1)(1,1)
   \pgfmathparse{sqrt(3)/2}
   \let\Mx\pgfmathresult
   \pgfmathparse{1/2}
   \let\My\pgfmathresult
   \tkzPoint[pos   = right,%
                  coord,%
            name  =%
            $M\left(\dfrac{\sqrt{3}}{2}~;~\dfrac{1}{2}\right)$](\Mx,\My){M}%
   \tkzSegment[style=dashed](O/M)
\end{tikzpicture}
```

An other test with `fp.sty` but now it's easy to make the same thing with Tikz. (test3.tex)

`fp.sty` is used to calculate the coordinates of the point M given by $\widehat{x\mathrm{OM}} = 1\mathrm{rad}$.

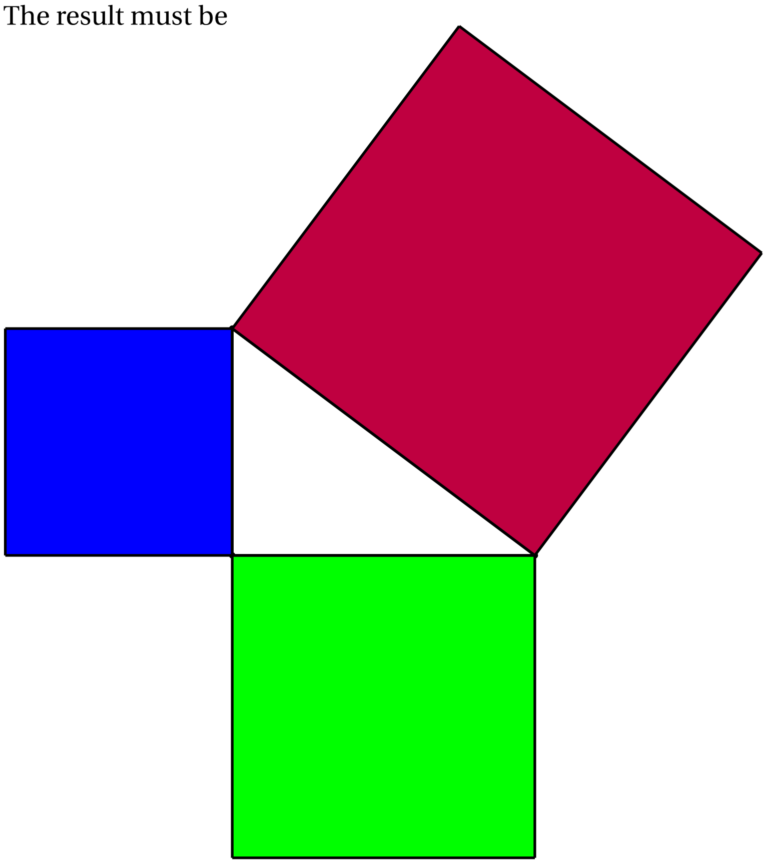



```
\begin{tikzpicture}
   \tkzInit[xmin=-1,xmax=1.2,xstep=.2,ymin=-1,ymax=1.2,ystep=.2]
   \tkzX[gradsize=\scriptstyle]\tkzY[gradsize=\scriptstyle]
   \tkzPoint(0,0){O}\tkzPoint[pos=above right](1,0){A}
   \FPcos\Mx{1}\FPsin\My{1}
   \tkzPoint[pos=above right](1,1){T}
   \tkzPoint[coord,mark = *,size = 1pt,%
            pos = above right](\Mx,\My){M}
   \tkzSegment[color=red,colorlabel=red,label=1\,u](A/T,O/M)
   \draw[color=blue] (0,0) circle (5cm);
   \path (A) arc (0:40:5) node[rotate=-45,above,color=red] {1\,u};
   \begin{scope}
      \path[clip](O)--(A)--(M)--cycle;
      \draw[color=blue,fill=red] (0,0) circle (.5cm);
   \end{scope}
   \begin{scope}
      \path[clip](O)--(A)--(T)--(M)--cycle;
      \draw[color=red] (0,0) circle (5cm);
   \end{scope}
   \path[clip] (0,0) circle (5cm);
   \tkzGrid(-1,-1)(1,1)
   \tkzText[color= red](0.3,0.15){$1$\,rad}
   \tkzText[style={draw},color= red](0.55,-0.15){$\scriptstyle\cos(1)$}
   \tkzText[style={draw},color= red](-0.23,0.83){$\scriptstyle\sin(1)$}
\end{tikzpicture}
```

Partie B Test of Tkz-2d The code below is the code of the file test4.tex

```
\documentclass{article}
\usepackage{amsmath}
\usepackage{tikz,tkz-2d}
\usetikzlibrary{arrows,plotmarks}%
\usepackage[english]{babel}
  % pour les français \usepackage[frenchb]{babel}
\usepackage[np,autolanguage]{numprint}
\begin{document}
\begin{tikzpicture}
   \tkzInit
   \tkzPoint[noname](0,0){C}
   \tkzPoint[noname](4,0){A}
   \tkzPoint[noname](0,3){B}
   \tkzSquare*(B,A){E}{F}
   \tkzSquare*(A,C){G}{H}
   \tkzSquare*(C,B){I}{J}
   \tkzFillPolygon[color = green ](A,C,G,H)
   \tkzFillPolygon[color = blue  ](C,B,I,J)
   \tkzFillPolygon[color = purple](B,A,E,F)
   \tkzPolygon[lw = 1pt](A,B,C)
   \tkzPolygon[lw = 1pt](A,C,G,H)
   \tkzPolygon[lw = 1pt](C,B,I,J)
   \tkzPolygon[lw = 1pt](B,A,E,F)
\end{tikzpicture}
\end{document}
```

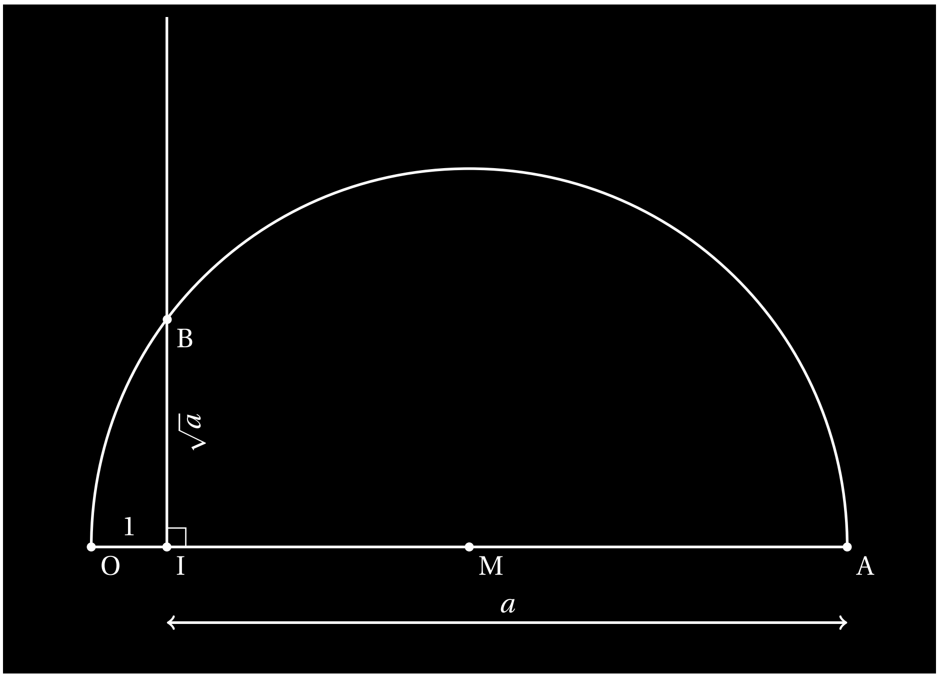
The result must be



III . Gallery : Some examples

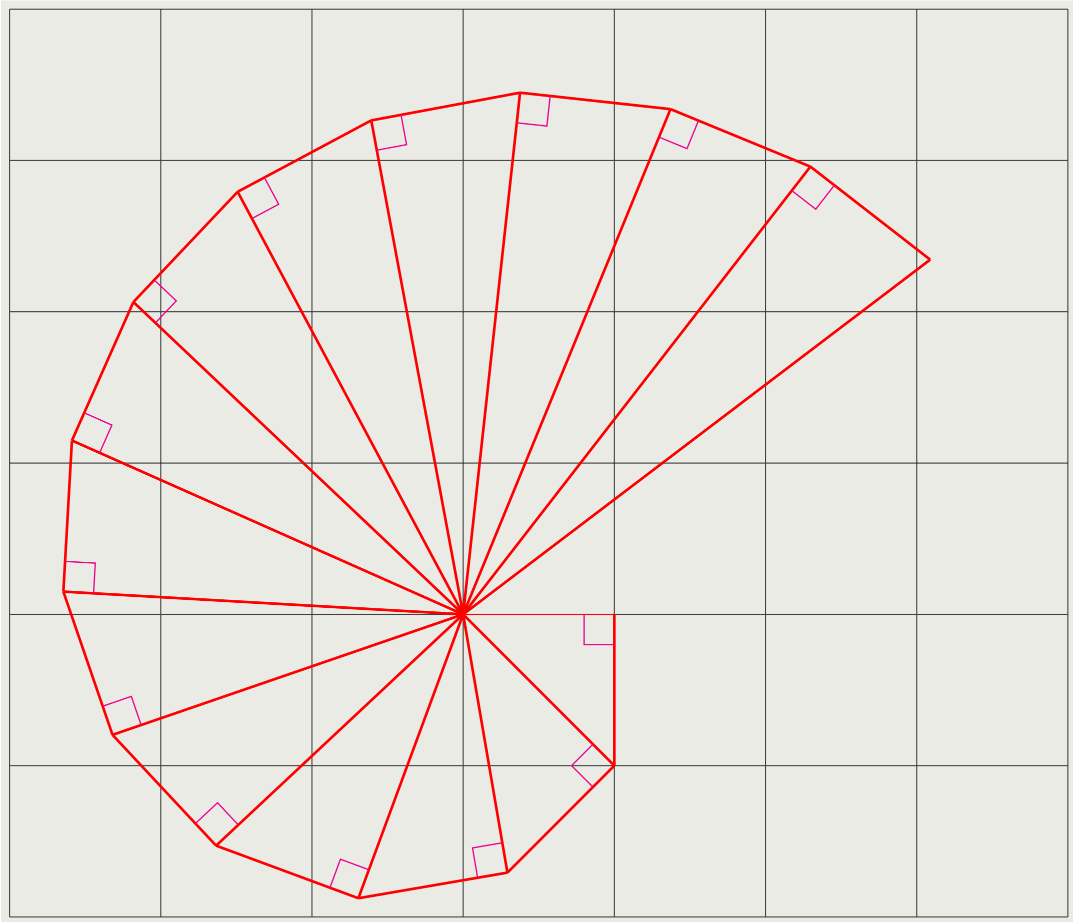
Example n° 1 White on Black

This example shows how to get a segment with a length equal at $\sqrt{a}$ from a segment of length a, only with a rule and a compass.



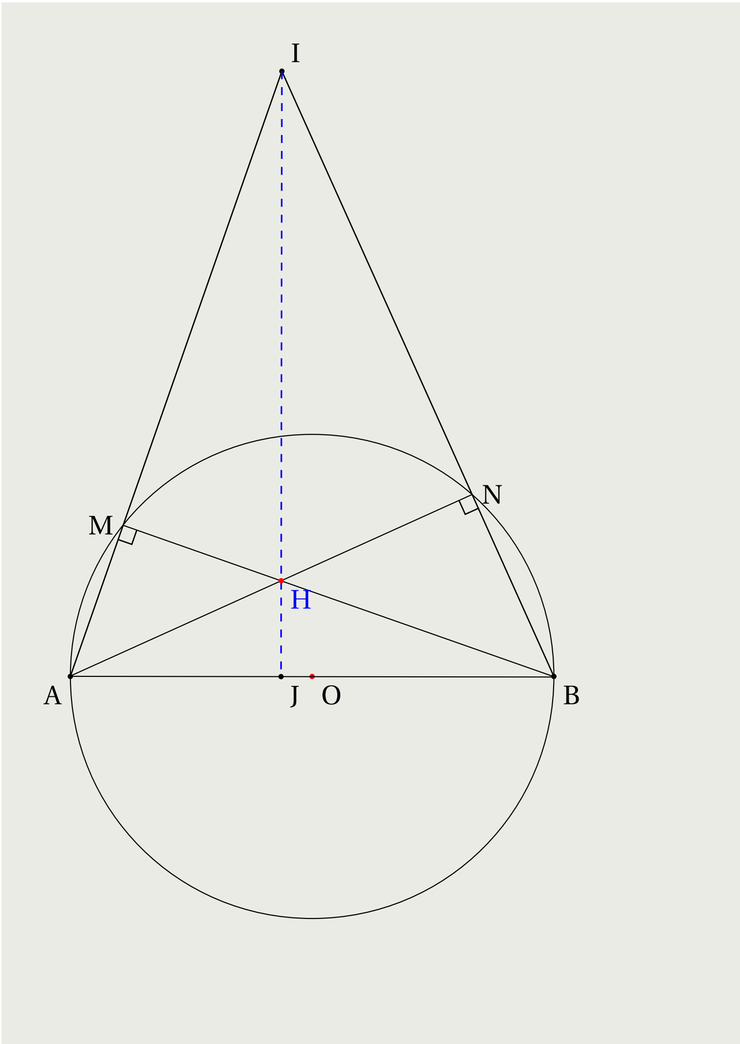

```
\begin{tikzpicture}[show background rectangle]
   \tkzInit[ymin=-1.5,ymax=7,xmin=-1,xmax=11]
   \tkzClip
   \tkzPoint*(0,0){O}
   \tkzPoint*(1,0){I}
   \tkzPoint*(1,-1){I'}
   \tkzPoint*(10,0){A}
   \tkzPoint*(10,-1){A'}
   \tkzVectorOrth(I,A){H}
   \tkzMidPoint*(O,A){M}
   \tkzInterLC(I,H)(M,M,A){C}{B}
   \tkzSegment[color=white,lw=1pt](I/H,O/A)
   \tkzDrawPoint[size      = 1.5pt,%
                 color     = white,%
                 namecolor = white](O,I,A,B,M)
   \tkzSegmentMark[colorlabel = white,%
                   poslabel   = -18pt,%
                   label      = $\sqrt{a}$](I/B)
   \tkzSegmentMark[colorlabel = white,%
                   label      = $1$](O/I)
   \tkzArc*[color=white,lw=1pt](M,M,A)(A,O)
   \draw[<->,color=white,line width=1pt]%
        (I') to node[midway,sloped,above,color=white]{$a$}(A');
   \tkzRightAngle[color=white](A/I/B)
\end{tikzpicture}
```

Example n° 2 How to get 1, $\sqrt{2}$, $\sqrt{3}$ with a rule and a compass.



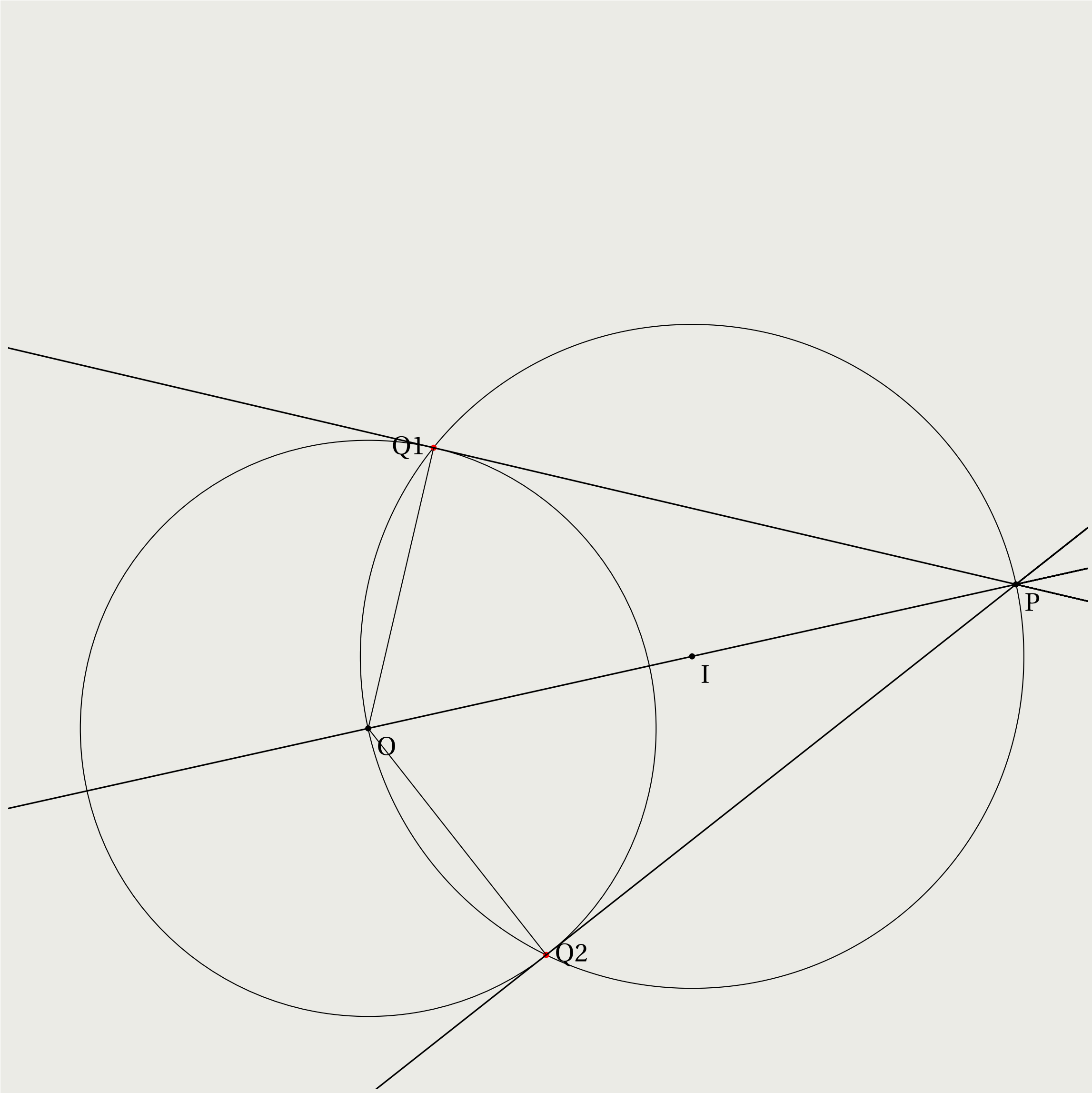
```
\begin{tikzpicture}[scale=2]
   \tkzInit[xmin=-3,xmax=4,ymin=-2,ymax=4]
   \tkzGrid
   \tkzPoint*(0,0){O}
   \tkzPoint*(1,0){a0}
   \tkzSegment[color=red](O/a0)
   \newcounter{tkzcounter}
   \setcounter{tkzcounter}{0}
   \foreach \i in {0,...,13}{%
      \stepcounter{tkzcounter}
      \tkzVectorOrthNormalised(a\i,O){a\thetkzcounter}
      \tkzPolySeg[color=red,lw=1pt](a\i,a\thetkzcounter,O)
      \tkzRightAngle[size=0.2,color=magenta](O/a\i/a\thetkzcounter)%
      }
 \end{tikzpicture}
```

Example n° 3 In this example, we want to shaw that points I,H and J are on the the same straight line.



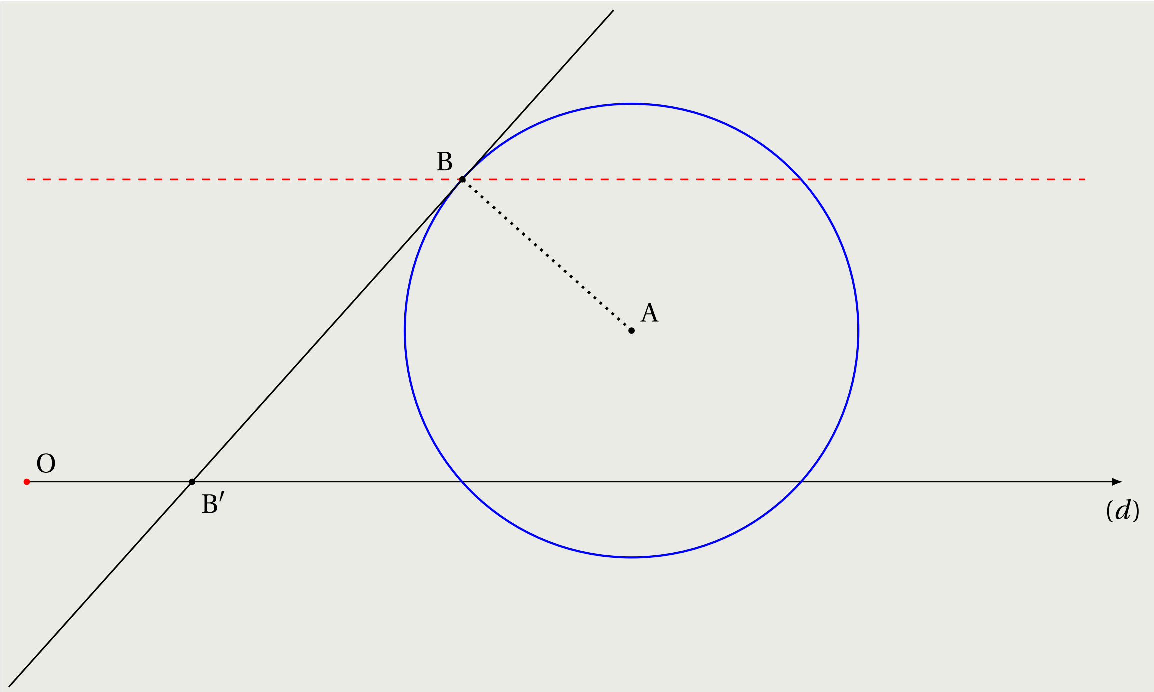

```
\begin{tikzpicture}[scale=.8]
  \tkzInit[ymin=-5,ymax=10]
  \tkzClip[space=1]
  \tkzPoint[pos=below left](0,0){A}
  \tkzPoint(8,0){B}
  \tkzMidPoint[color=red](A,B){O}
  \tkzCircle*(A,B)
  \tkzPoint[pos=above right](3.5,10){I}
  \tkzSegment[lw=.5pt](I/A,I/B)
  \tkzInterLCR(I,A)(O,4 cm){A}{M}
  \tkzInterLCR(I,B)(O,4 cm){N}{B}
  \tkzDrawPoint[mark={},pos=left](M)
  \tkzDrawPoint[mark={},pos=right](N)
  \tkzSegment(A/B,B/M,A/N)
  \tkzRightAngle(A/M/B,A/N/B)
  \tkzProjection(A,B)(I/J)
  \tkzSegment[style=dashed,color=blue,lw=0.6pt](I/J)
  \tkzInterLL[color=red,namecolor=blue](A,N)(B,M){H}
\end{tikzpicture}
```

Example n° 4 How to construct the tangent lines from a point to a circle with a rule and a compass.




```
\begin{tikzpicture}
  \tkzInit[xmin=-5,ymin=-5]
  \tkzClip
  \tkzPoint(0,0){O}\tkzPoint(9,2){P}
  \tkzMidPoint(O,P){I}
  \tkzCircleR(O,4cm)
  \tkzCircle*(O,P)
  \tkzMathLength(I,P)
  \tkzInterCCR(O,4cm)(I,\tkzmathLen pt){Q1}{Q2}
  \tkzDrawPoint[mark=*,size=1pt,color=red,pos=left](Q1)
  \tkzDrawPoint[mark=*,size=1pt,color=red,pos=right](Q2)
  \tkzLine(P/Q1,P/Q2,P/O)
  \tkzSegment(O/Q1,O/Q2)
\end{tikzpicture}
```

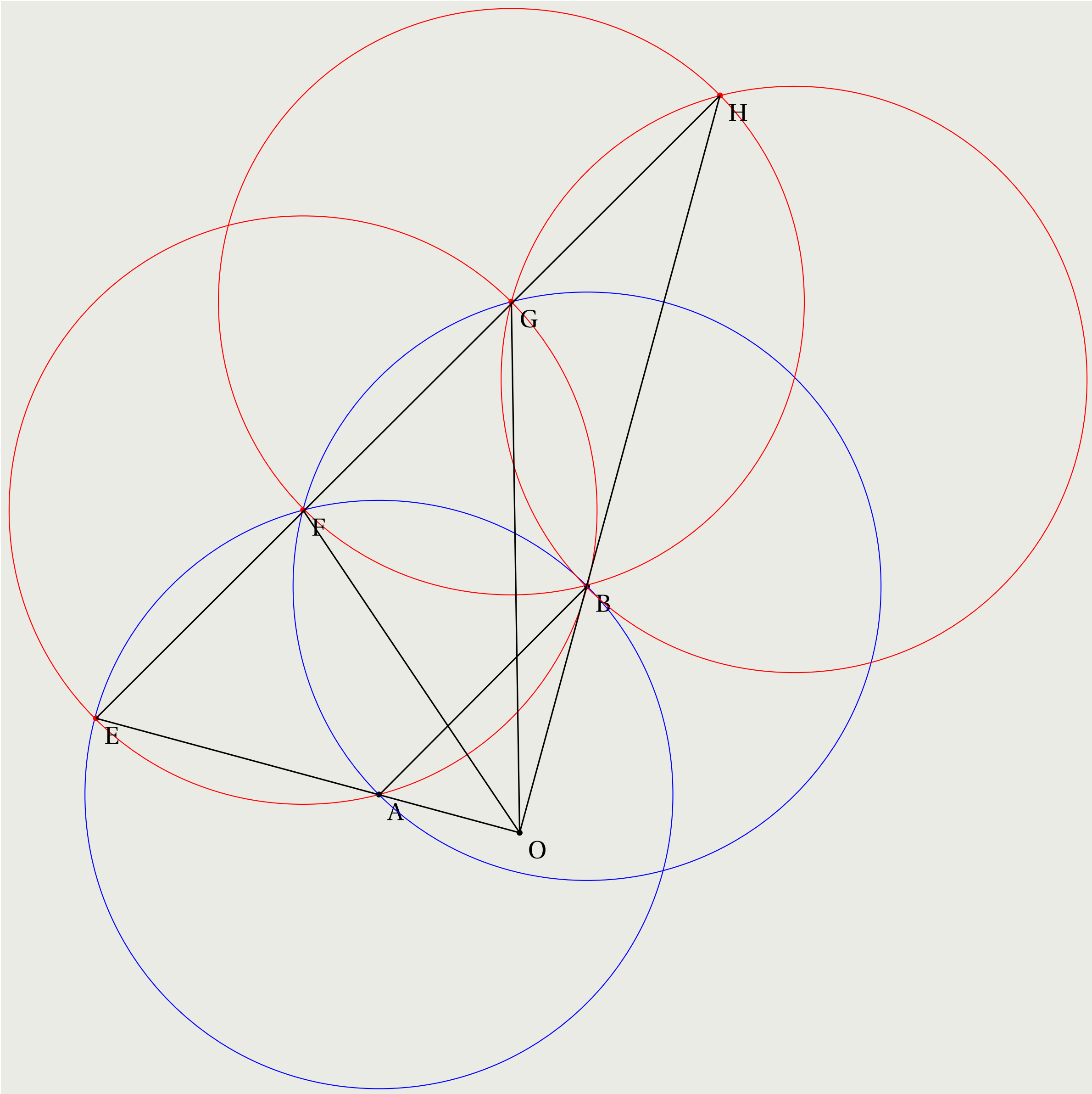
Example n° 5 Circle and tangent




```
\begin{tikzpicture}
   \tkzInit[xmax=14,ymin=-2,ymax=6]
   \tkzX[noticks,nograd,label=$(d)$]
   \tkzPoint[pos=above right](8,2){A};
   \tkzPoint[color=red,pos=above right](0,0){O};
   \draw[blue,line width=.8pt]  (A) circle(3cm);%todo circle r=
   \tkzHLine[color=red,style=dashed]{4}
   \pgfmathparse{8-3*cos(asin(2/3))}
   \let\xB\pgfmathresult
   \tkzPoint[pos=above left](\xB,4){B};
   \tkzSegment[style=dotted,lw=1pt](A/B)
   \tkzLineOrth[kr=3,kl=1,prefix=t](A,B)(B)
   \tkzPoint*(1,0){i}
   \tkzInterLL(tl,tr)(O,i){B'}
  \end{tikzpicture}
```

Example n° 6 How to divide a segment in three segments of the same length

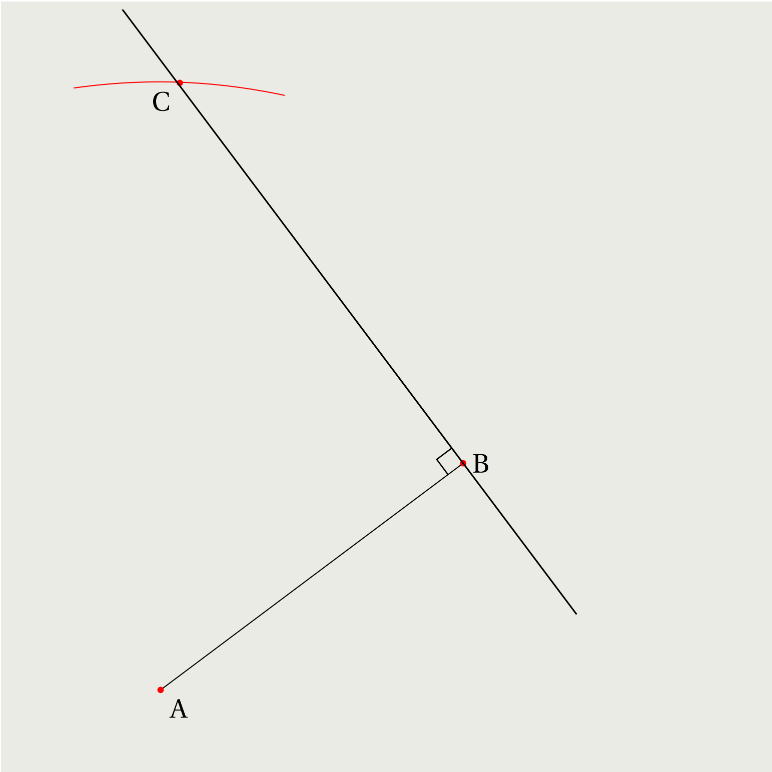
The diagram below shows how to trisect a line segment [AB]




```
\begin{tikzpicture}
\tkzInit
\tkzPoint(0,0){A} \tkzPoint(3,3){B}
\tkzCircleR*[color=blue](B,B,A)
\tkzCircleR*[color=blue](A,A,B)
\tkzInterCC(B,B,A)(A,A,B){A2}{F}
\tkzCircleR*[color=red](F,A,B)
\tkzInterCC(F,A,B)(B,B,A){G}{A}
\tkzInterCC(F,A,B)(A,A,B){B}{E}
\tkzCircleR*[color=red](G,A,B)
\tkzInterCC(G,A,B)(B,B,A){A7}{A8}
\tkzCircleR*[color=red](A7,A,B)
\tkzInterCC(A7,A,B)(G,B,A){A9}{H}
\tkzDrawPoint[color=red](F,G,E,H)
\tkzInterLL(E,A)(H,B){O}
\tkzSegment[lw=.6 pt](A/B,O/H,O/E,E/H,F/O,G/O)
\end{tikzpicture}
```

Example n° 7 About right triangle

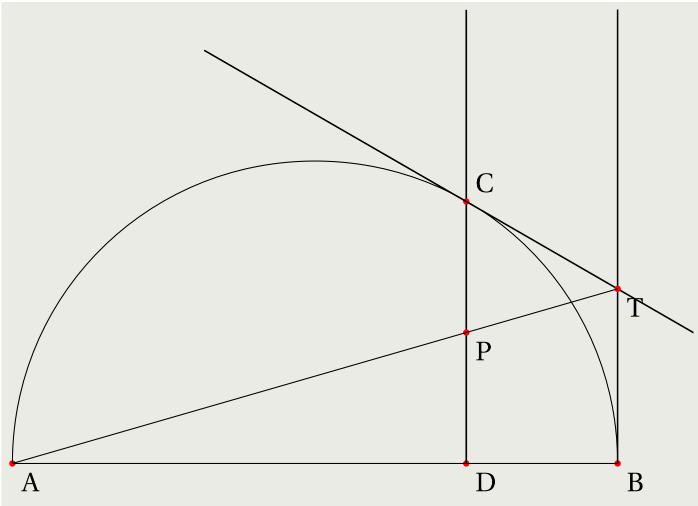
We have a segment [AB] and we want to determine a point C such as AC = $8cm$ and ABC is a right triangle in B.




```
\begin{tikzpicture}
  \tkzInit
  \tkzClip
  \path[coordinate] (2,1) coordinate(A)%
                    (6,4) coordinate(B);
  \tkzSegment(A/B)
  \tkzDrawPoint[color=red](A)
  \tkzDrawPoint[color=red,pos=right](B)
  \tkzVectorOrth*(B,A){H}
  \tkzInterLCR(B,H)(A,8 cm){C}{J}
  \tkzDrawPoint[color=red,pos=below left](C)
  \tkzCompass[color=red](A,C)
  \tkzLineOrth[kr=3,kl=.5](A,B)(B)
  \tkzRightAngle(A/B/C)
\end{tikzpicture}
```

Example n° 8 Archimedes

This is an ancient problem proved by the great Greek mathematician Archimedes . The figure below shows a semicircle, with diameter AB. A tangent line is drawn and touches the semicircle at B. An other tangent line at a point, C, on the semicircle is drawn. We project the point C on the segment[AB] on a point D . The two tangent lines intersect at the point T.
Prove that the line (AT) bisects (CD)




```
\begin{tikzpicture}
  \tkzInit[ymin=-1]
  \path[coordinate] (0,0) coordinate(A)%
                    (6,0) coordinate(D)
                    (8,0) coordinate(B)
                    (4,0) coordinate(I);
  \tkzDrawPoint[color=red](A,B,D)
  \tkzSegment(A/B)
  \clip (A)--(9,0)--(9,6)--(0,6)--cycle;
  \tkzCircle*(A,B)
  \tkzLineOrth[kr=1,kl=0](A,D)(D)
  \tkzInterLCR(D,dr)(I,4 cm){C}{J}
  \tkzDrawPoint[color=red,pos=above right](C)
  \tkzLineOrth[kr=1,kl=1,prefix=t1](I,C)(C)
  \tkzLineOrth[kr=1,kl=0,prefix=t2](A,B)(B)
  \tkzInterLL[color=red](C,t1r)(B,t2r){T}
  \tkzInterLL[color=red](A,T)(C,D){P}
  \tkzSegment(A/T)
\end{tikzpicture}
```

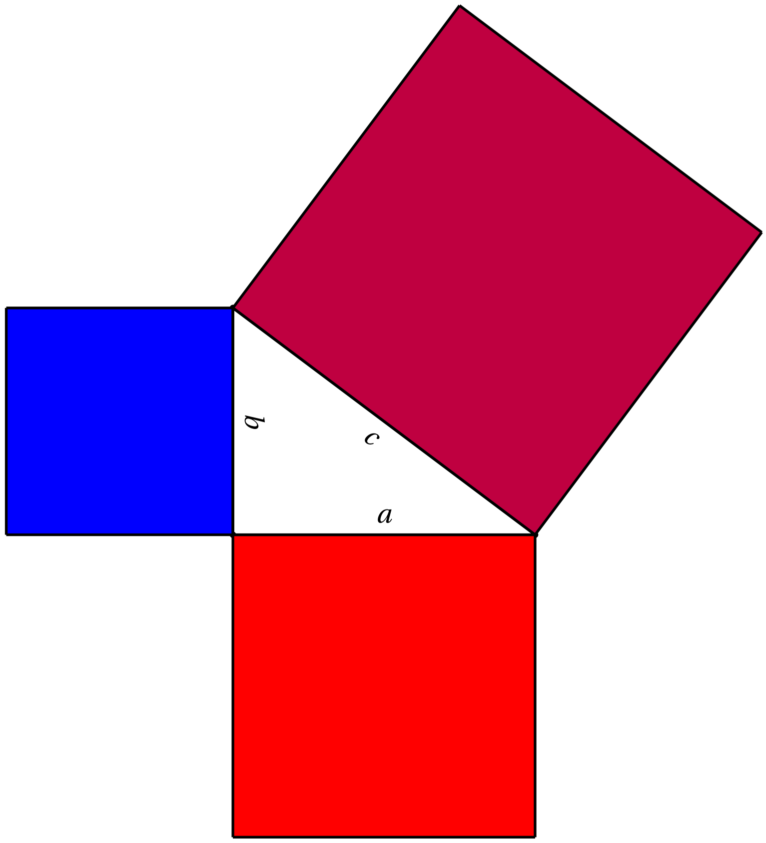
IV . Gallery : Pythagoras' Theorem

The most well-known theorem in Euclid Geometry is about the right-angle triangle, commonly attributed to Pythagoras but Greek, Chinese and Babylonian mathematicians have known about this for a long time. The theorem is of fundamental importance in Euclidean Geometry where it serves as a basis for the definition of distance between two points.

The Pythagoras' theorem

In any right angled triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides other than the hypotenuse).

Example n° 9 The Pythagoras' theorem



```
\begin{tikzpicture}
   \tkzInit
   \tkzPoint[noname](0,0){C}
   \tkzPoint[noname](4,0){A}
   \tkzPoint[noname](0,3){B}
   \tkzSquare*(B,A){E}{F}
   \tkzSquare*(A,C){G}{H}
   \tkzSquare*(C,B){I}{J}
   \tkzFillPolygon[color  =  red   ](A,C,G,H)
   \tkzFillPolygon[color  =  blue  ](C,B,I,J)
   \tkzFillPolygon[color  =  purple](B,A,E,F)
   \tkzPolygon[lw  =  1pt](A,B,C)
   \tkzPolygon[lw  =  1pt](A,C,G,H)
   \tkzPolygon[lw  =  1pt](C,B,I,J)
   \tkzPolygon[lw  =  1pt](B,A,E,F)
   \tkzSegmentMark[label  =  $a$](C/A)
   \tkzSegmentMark[label  =  $b$](B/C)
   \tkzSegmentMark[label  =  $c$,poslabel  =  -14pt](B/A)
\end{tikzpicture}
```

In this picture, the area of the blue square added to the area of the red square makes the area of the purple square.

This is usually summarised as:

The square on the hypotenuse is equal to the sum of the squares on the other two sides.

If we let c be the length of the hypotenuse and a and b be the lengths of the other two sides, the theorem can be expressed as the equation

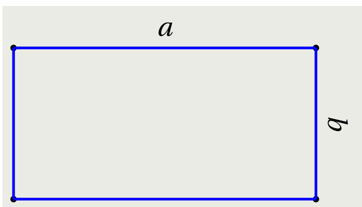
$$a^2 + b^2 = c^2$$

or, solved for c:

$$c = \sqrt{a^2 + b^2}.$$

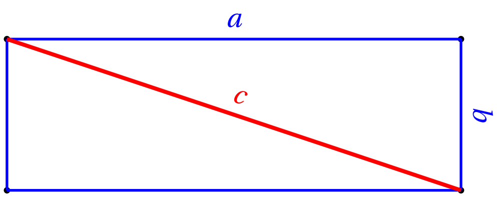
Example n° 10 Modern proof by algebra

Draw a rectangle with two sides of a and b (say the lower left).




```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint[noname](-1,2){A} \tkzPoint[noname]( 3,4){C}
  \tkzPoint[noname]( 3,2){B} \tkzPoint[noname](-1,4){D}
  \tkzSegment[label  =  $a$,color  =  blue,lw  =  1pt](C/D)
  \tkzSegment[label  =  $b$,color  =  blue,lw  =  1pt](C/B)
  \tkzSegment[color  =  blue,lw  =  1pt](D/A,A/B)
\end{tikzpicture}
```

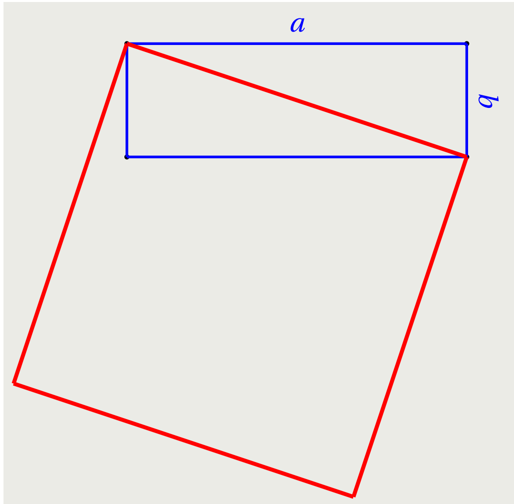
Draw a diagonal of this rectangle (call it c).



We add this code

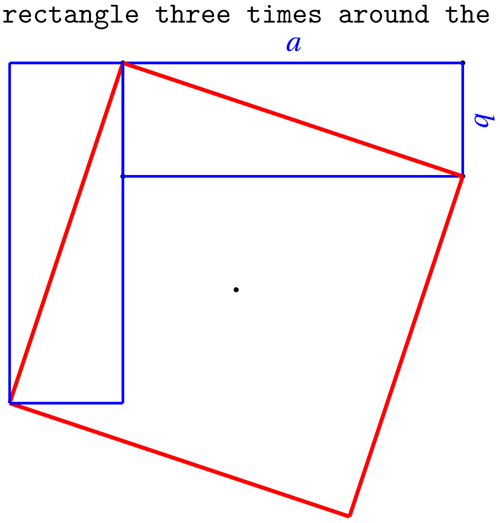
```
\tkzSegment[label = $c$,colorlabel = red,color = red,lw = 1.5pt](D/B)
```

Draw a square using the diagonal as its side (shown in red).



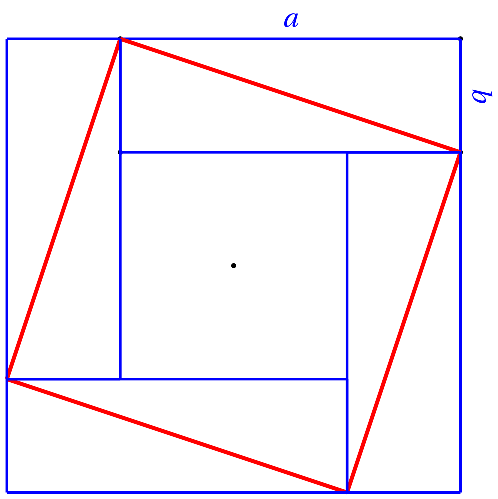

```
\begin{tikzpicture}[scale=.75]
   \tkzInit
   \tkzPoint[noname](-1,2){A} \tkzPoint[noname]( 5,4){C}
   \tkzPoint[noname]( 5,2){B} \tkzPoint[noname](-1,4){D}
   \tkzSegment[label = $a$,colorlabel = blue,color = blue,lw = 1pt](C/D)
   \tkzSegment[label = $b$,colorlabel = blue,color = blue,lw = 1pt](C/B)
   \tkzSegment[color = blue,lw = 1pt](D/A,A/B)
   \tkzSquare*(B,D){I}{J}
   \tkzPolygon[color = red,lw = 1.5pt](B,D,I,J)
\end{tikzpicture}
```

Replicate the original rectangle three times around the red square.



We add this code

```
   \tkzRotate(O,90)(C/C1,D/D1,A/A1,B/B1)
   \tkzPolygon[color = blue,lw = 1pt](A1,B1,C1,D1)
```



We have a square of side $a+b$, inscribed in it is a red square of side c, and four copies of the original right triangle.

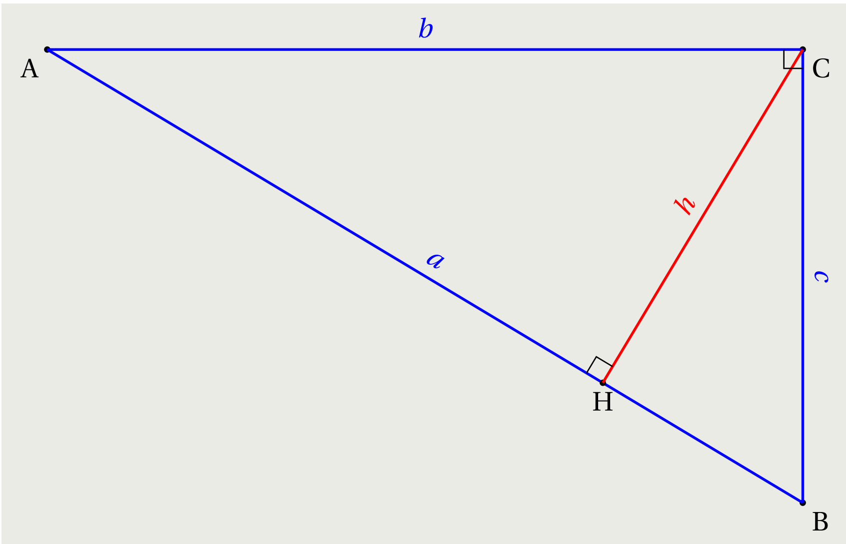
$$\begin{aligned}4\frac{ab}{2}+c^2&=(a+b)^2\\2ab+c^2&=a^2+2ab+b^2\\c^2&=a^2+b^2\end{aligned}$$

Another possibilty
Area of red square = Areas of 4 triangles + Area of small square.
The algebra here asserts that

$$\begin{aligned}4\frac{ab}{2}+(b-a)^2&=c^2\\2ab+b^2-2ab+a^2&=c^2\\a^2+b^2&=c^2\end{aligned}$$

Example n° 11 Proof using similar triangles

Like many of the proofs of the Pythagoras' Theorem, this one is based on the proportionality of the sides of two similar triangles.
Let ABC represents a right triangle, with the right angle located at C, as shown on the figure.



```
\begin{tikzpicture}
  \tkzInit[ymax=6]\tkzClip[space=0.5]
  \tkzPoint(10,6){C}
  \tkzPoint[pos=below left]( 0,6){A}
  \tkzPoint(10,0){B}
  \SetUpSegment[colorlabel=blue,color=blue,lw=1pt]
  \tkzSegment[label=$c$](C/B)
  \tkzSegment[label=$b$](A/C)
  \tkzSegment[label=$a$](A/B)
  \tkzProjection[pos=below](B,A)(C/H)
  \tkzSegment[label=$h$,colorlabel=red,color=red](C/H)
  \tkzRightAngle(B/C/A,C/H/A)
\end{tikzpicture}
```

We draw the altitude from point A, and call H its intersection with the side AB. The new triangle ACH is similar to the triangle ABC, because they both have a right angle (by definition of the altitude), and they share the angle at A. The triangle BCH is also similar to ABC. The two similarities lead to the two ratios:

$$\frac{a}{c} = \frac{\mathrm{HB}}{a} \text{ and } \frac{b}{c} = \frac{\mathrm{AH}}{b}$$

These can be written as

$$a^2 = c \times \mathrm{HB} \text{ and } b^2 = c \times \mathrm{AH}.$$

Summing these two equalities, we obtain

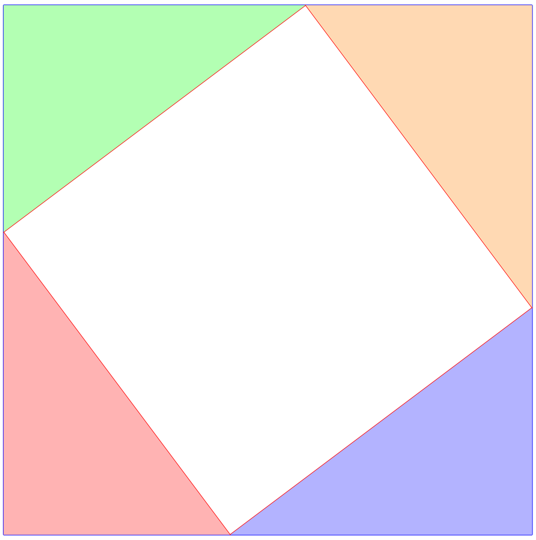
$$a^2 + b^2 = c \times \mathrm{HB} + c \times \mathrm{AH} = c \times (\mathrm{HB} + \mathrm{AH}) = c^2.$$

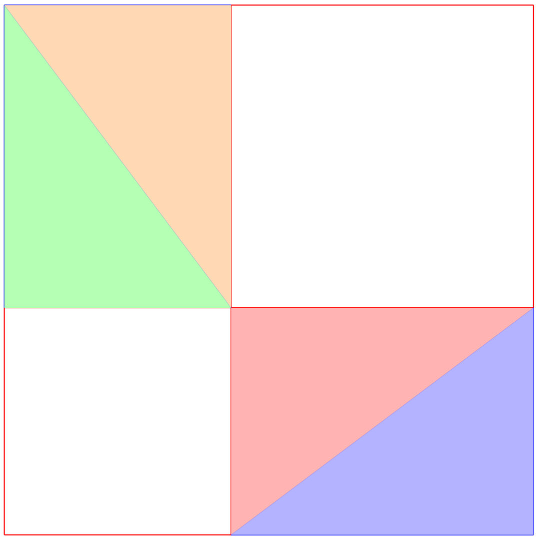
In other words, the Pythagoras' Theorem:

$$a^2 + b^2 = c^2$$

Example n° 12 Proof by rearrangement

The area of each large square is $(a+b)^2$. In both figures, the area of four identical triangles is removed. The remaining areas in white, a^2+b^2 and c^2, are equal.



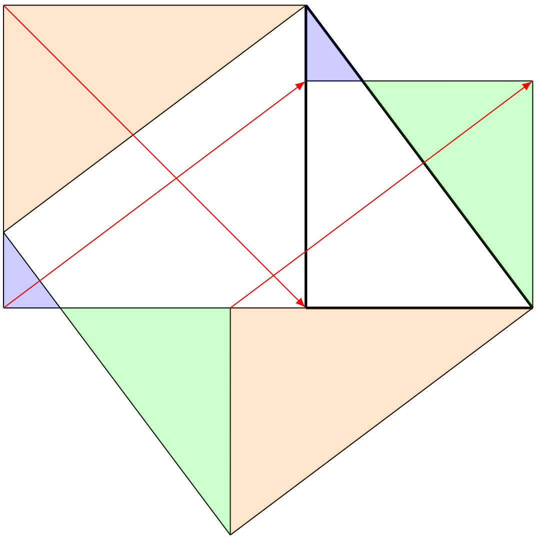


```
\begin{tikzpicture}
   \tkzInit
   \tkzPoint[noname](0,0){A} \tkzPoint[noname](7,0){B}
   \tkzSquare*(A,B){C}{D}
   \tkzPolygon[color = blue](B,C,D,A)%
   \tkzPoint[noname](3,0){I} \tkzPoint[noname](7,3){J}
   \tkzSquare*(I,J){K}{L}
   \tkzPolygon[color = red](I,J,K,L)%
   \tkzFillPolygon[color = red!30](A,I,L)%
   \tkzFillPolygon[color = blue!30](I,B,J)%
   \tkzFillPolygon[color = orange!30](K,C,J)%
   \tkzFillPolygon[color = green!30](K,D,L)%
\end{tikzpicture}
```

```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint[noname](0,0){A} \tkzPoint[noname](7,0){B}
  \tkzSquare*(A,B){C}{D}
  \tkzPoint[noname](3,0){I} \tkzPoint[noname](7,3){J}
  \tkzSquare*(A,I){M}{L}
  \tkzSquare*(M,J){C}{P}
  \tkzPolygon[color = red](A,I,M,L)%
  \tkzPolygon[color = red](M,J,C,P)%
  \tkzSegment[color = blue](I/J,D/M,I/B,B/J,P/D,D/L)
  \tkzFillPolygon[color = red!30](M,J,I)%
  \tkzFillPolygon[color = blue!30](I,B,J)%
  \tkzFillPolygon[color = orange!30](D,P,M)%
  \tkzFillPolygon[color = green!30](D,M,L)%
\end{tikzpicture}
```

Example n° 13 Proof using area subtraction

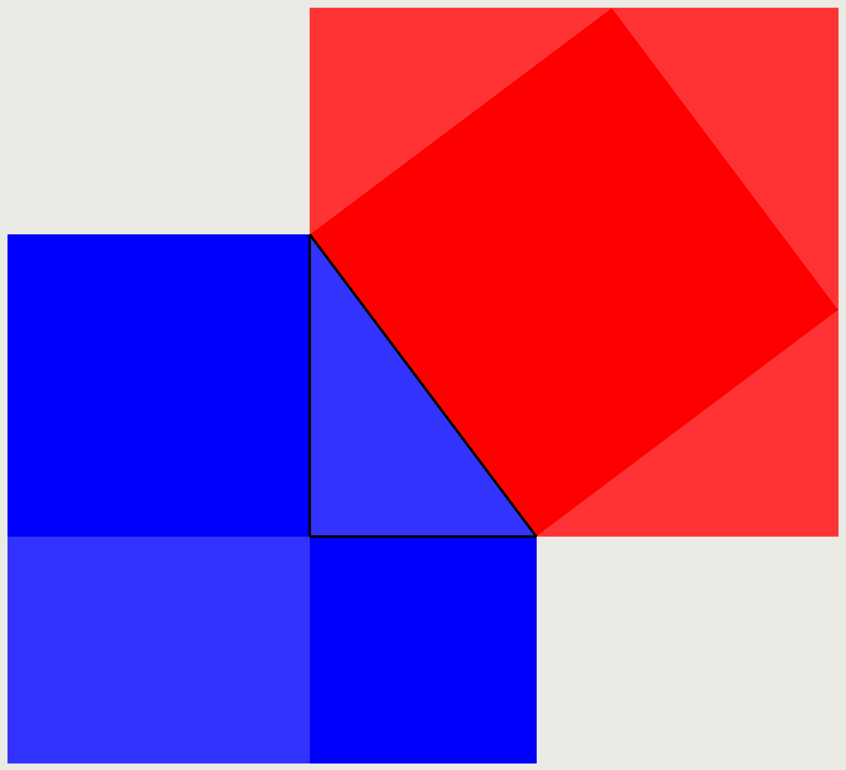
The red arrows show three translations of triangles.



```
\begin{tikzpicture}
   \tkzInit
   \tkzPoint*(0,0){A} \tkzPoint*(4,0){B}
   \tkzSquare*(A,B){C}{D}
   \tkzPoint*(4,3){I}
   \tkzSquare*(I,B){J}{K}
   \tkzSquare*(J,C){L}{M}
   \tkzPolygon[lw = 1pt](B,C,J)%
   \tkzProjection*(A,B)(M/T)
   \tkzSegment[](K/J,I/K,L/M,M/J,C/L,C/D,D/A,A/B,M/T)
   \tkzInterLL*(A,B)(L,M){R}
   \tkzInterLL*(I,K)(C,J){S}
   \tkzFillPolygon[color = blue,opacity = .2](S,C,I)%
   \tkzFillPolygon[color = blue,opacity = .2](R,L,A)%
   \tkzFillPolygon[color = orange,opacity = .2](D,C,L)%
   \tkzFillPolygon[color = orange,opacity = .2](T,J,M)%
   \tkzFillPolygon[color = green,opacity = .2](J,K,S)%
   \tkzFillPolygon[color = green,opacity = .2](T,R,M)%
   \tkzDrawVector[color = red](D/B,A/I,T/K)
\end{tikzpicture}
```

Example n° 14 Proof using area comparaison

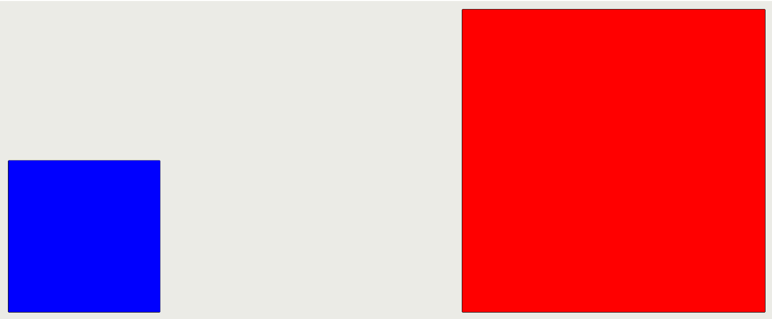
This is a variant of the above proof. Two regions - the red and the blue - are equal.



```
\begin{tikzpicture}
   \tkzInit
   \tkzPoint*(0,0){A} \tkzPoint*(3,0){B} \tkzPoint*(0,4){C}
   \tkzSquare*(B,A){D}{E}
   \tkzSquare*(A,C){F}{G}
   \tkzSquare*(C,B){H}{I}
   \path[fill = blue,opacity = 1] (E) rectangle (F);
   \path[fill = blue!80,opacity = 1] (D)-|(G)--(A)--cycle;
   \path[fill = blue!80] (A)--(B)--(C)--cycle;
   \path[fill = red!80] (B)-|(H)--(B)--cycle;
   \path[fill = red!80] (H)|-(I)--cycle;
   \path[fill = red!80] (I)-|(C)--cycle;
   \tkzFillPolygon[color = blue](B,A,D,E)
   \tkzFillPolygon[color = blue](A,C,F,G)
   \tkzFillPolygon[color = red,opacity = 1](C,B,H,I)
   \tkzPolygon[lw = 1pt](A,B,C)
\end{tikzpicture}
```

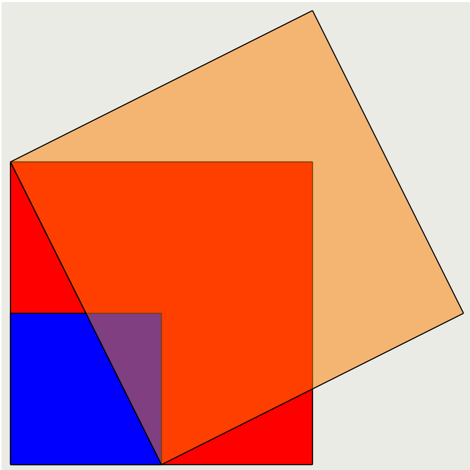
Example n° 15 Sum of two squares

We start with two squares with sides a and b, respectively, placed side by side. The total area of the two squares is $a^2 + b^2$.



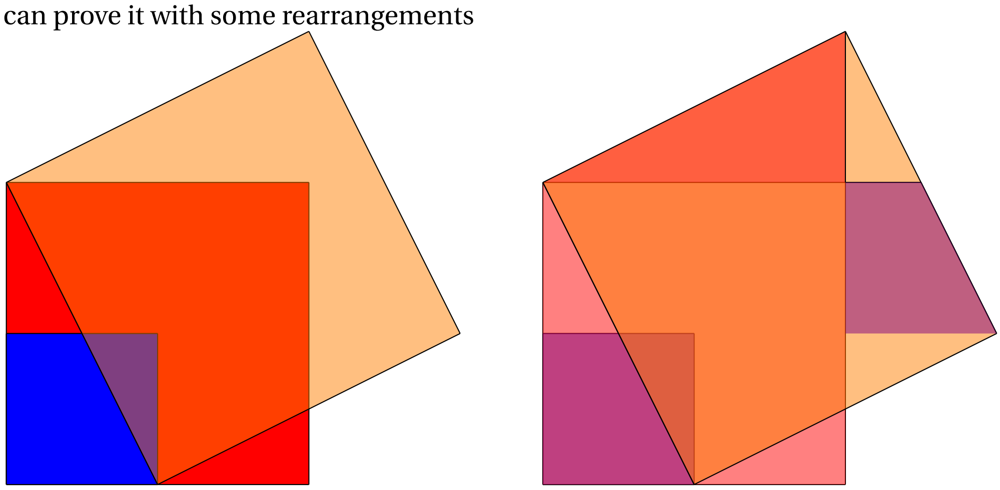
```
\begin{tikzpicture}[scale = 2]
  \tkzInit
  \tkzPoint*(0,0){A} \tkzPoint*(0,1){B}
  \tkzSquare*(A,B){C}{D}
  \tkzPoint*(2,0){I}\tkzPoint*(4,0){J}
  \tkzSquare*(I,J){K}{L}
  \tkzPolygon(A,B,C,D)%
  \tkzPolygon(I,J,K,L)%
  \tkzFillPolygon[color = blue](A,B,C,D)%
  \tkzFillPolygon[color = red](I,J,K,L)%
\end{tikzpicture}
```

The aera of the orange square is $a^2 + b^2$



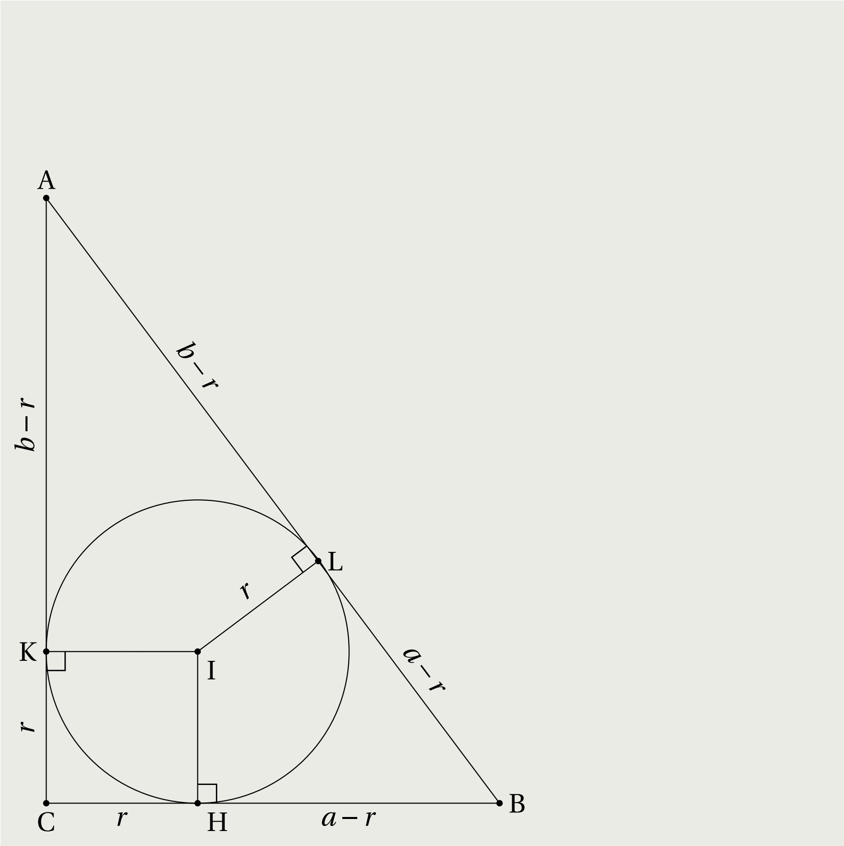
```
\begin{tikzpicture}[scale = 2]
   \tkzInit
   \tkzPoint*(0,0){I'}\tkzPoint*(2,0){J'}
   \tkzPoint*(1,0){B'}
   \tkzSquare*(I',J'){K'}{L'}
   \tkzFillPolygon[color = red](I',J',K',L')%%
   \tkzPolygon(I',J',K',L')%
   \tkzSquare*(I',B'){C'}{D'}
   \tkzFillPolygon[color = blue](I',B',C',D')%
   \tkzSegment(L'/B')
   \tkzSquare*(L',B'){M}{N} \tkzPolygon(I',B',C',D')
   \tkzFillPolygon[color   = orange,%
                   opacity = .5](L',B',M,N)%
   \tkzPolygon(L',B',M,N)%
\end{tikzpicture}
```

Now, we can prove it with some rearrangements



```
\begin{tikzpicture}[scale = 2]
  \tkzInit
  \tkzPoint*(0,0){I'}%
  \tkzPoint*(2,0){J'}%
  \tkzPoint*(1,0){B'}
  \tkzSquare*(I',J'){K'}{L'}
  \tkzSquare*(I',B'){C'}{D'}
  \tkzSquare*(L',B'){M}{N}%
  \tkzPolygon(I',B',C',D')%
  \tkzPolygon(I',J',K',L')%
  \tkzFillPolygon[color   = blue,%
                  opacity = .5](I',B',C',D')%
  \tkzFillPolygon[color   = red,%
                  opacity = .5](I',J',K',L')%
  \tkzFillPolygon[color   = orange,%
                  opacity = .5](L',B',M,N)%
  \tkzFillPolygon[color   = red,%
                  opacity = .5](K',N,L')%
  \tkzPolygon(L',B',M,N)%
  \tkzInterLL*(L',K')(M,N){S}
  \tkzInterLL*(D',C')(J',K'){T}
  \tkzInterLL*(B',M)(J',K'){U}
  \tkzSegment(L'/B',K'/N,S/M,K'/S)
  \tkzFillPolygon[color   = blue!50!red,%
                  opacity = .5](T,M,S,K')%
\end{tikzpicture}
```

Example n° 16 Proof with InCircle




```
\begin{tikzpicture}
  \tkzInit \tkzClip[space=.5]
  \tkzPoint[pos = below](0,0){C}\tkzPoint[pos = right](6,0){B}%
  \tkzPoint[pos = above](0,8){A}
  \tkzPolygon(A,B,C)%%
  \tkzInCenter(A,B,C){I}
  \tkzProjection(B,C)(I/H)
  \tkzProjection[pos = left](A,C)(I/K)
  \tkzProjection[pos = right](A,B)(I/L)
  \tkzSegment(I/L,I/H,I/K)
  \tkzCircle(I,H)
  \tkzRightAngle(A/L/I,B/H/I,C/K/I)
  \tkzSegmentMark[label = $r$,poslabel = -12pt](C/H)
  \tkzSegmentMark[label = $a-r$,poslabel = -12pt](H/B)
  \tkzSegmentMark[label = $r$](C/K)
  \tkzSegmentMark[label = $b-r$](K/A)
  \tkzSegmentMark[label = $a-r$](L/B)
  \tkzSegmentMark[label = $b-r$](A/L)
  \tkzSegmentMark[label = $r$](I/L)
\end{tikzpicture}
```

Let ABC represents a right triangle, with the right angle located at C, as shown on the figure. Let a, b and c the lengths of the three sides; c is the length of the hypothenuse.
Let r and s be the radius of the incercle and the semiperimeter of the triangle.
a, b and c can be regarded in relation to r and they may be expressed with r: $a = r+(a-r)$, $b = r+(b-r)$ and $c = (a-r)+(b-r)$.
In a right triangle, we have the relation $r = s-c$. From the diagram, the hypotenuse AB is split in two pieces: $(a-r)$ and $(b-r)$, the length of the hypothenuse is $c = (a-r)+(b-r)$.
The perimeter is a function of r

$$2s = a+b+c = r+(a-r)+r+(b-r)+(a-r)+(b-r) = 2a+2b-2r$$

so we can expressed r with s and c

$$2r = a + b - c = 2s - 2c \text{ and } r = s - c.$$

Compute the area of the triangle ABC in three different ways.

1/ ABC is a right triangle so,

$$\text{Area(ABC)} = \frac{1}{2}ab.$$

2/ The aera of a triangle is given by rs with $s = \frac{1}{2}(a + b + c)$.

$$\text{Area(ABC)} = \text{Area(AIC)} + \text{Area(CIB)} + \text{Area(BIA)}$$

The aera of triangle AIC is given by $\frac{1}{2} \times \text{AC} \times \text{IK} = \frac{1}{2}br$, therefore and by analogy :

$$\text{Area(ABC)} = \frac{1}{2}r(a + b + c) = rs$$

3/ In a right triangle, we have the relation $r = s - c$ (see above) so,

$$\text{Area(ABC)} = rs = s(s - c)$$

$$rs = (s - c)s = \left(\frac{a + b + c}{2} - c\right)\left(\frac{a + b + c}{2}\right) = \frac{(a + b - c)(a + b + c)}{4} = \frac{1}{2}ab$$

Simplifications yield

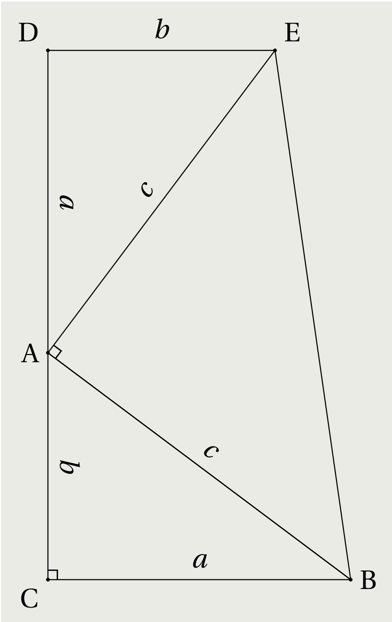
$$(a + b)^2 - c^2 = 2ab$$

and

$$a^2 + b^2 = c^2$$

Example n° 17 Proof with a trapezoid

This proof, discovered by President J.A. Garfield in 1876




```
\begin{tikzpicture}[scale=.5]
  \tkzInit
  \tkzPoint[pos = below left](0,0){C}\tkzPoint[pos = right](8,0){B}%
  \tkzPoint[pos = left](0,6){A}\tkzPoint[pos = above left](0,14){D}%
  \tkzPoint[pos = above right](6,14){E}
  \tkzPolygon(A,B,C)%%
  \tkzPolygon(A,D,E,B)%%
  \tkzSegment(E/A)
  \tkzRightAngle(A/C/B,B/A/E)
  \tkzSegmentMark[label = $c$](A/B)
  \tkzSegmentMark[label = $b$](A/C)
  \tkzSegmentMark[label = $a$](B/C)
  \tkzSegmentMark[label = $b$](D/E)
  \tkzSegmentMark[label = $a$](D/A)
  \tkzSegmentMark[label = $c$](A/E)
\end{tikzpicture}
```

The formula for the area of a trapezoid is the key, but this aera can be computed as the sum of areas of the three triangles ABC, ADE and EAB (EAB is an isosceles right triangle)

$$\frac{1}{2}(a+b)(a+b) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$$

Simplifications yield $a^2 + b^2 = c^2$.

V . Gallery : Some Sangaku Problems

Some references

- a Scientific American article by Tony Rothman written in co-operation with Hidetoshi Fukagawa;
- the book by H. Fukagawa and D. Pedoe.
- http://www.sangaku.info/
- http://mathworld.wolfram.com
- http://www.wasan.jp/english/
- http://www.cut-the-knot.org/pythagoras/Sangaku.shtml

What are Sangaku or San Gaku ?

Sangaku are colorful wooden tablets which were hung often in shinto shrines and sometimes in buddhist temples in Japan and posing typical and elegant mathematical problems. The problems featured on the sangaku are problems of japanese mathematics (wasan). The earliest sangaku found date back to the beginning of the 17th century.

Sangaku Two Unrelated Circles

Chord [ST] *is perpendicular to diameter* [CP] *of a circle with center* O *at point* R. Q *is point of* [CP] *between* P *and* R . [SQ] *intersects the circle in* V .
Let r *be the radius of the circle inscribed into the curvilinear triangle TQV. Prove that*

$$\frac{1}{r} = \frac{1}{\mathrm{PQ}} + \frac{1}{\mathrm{QR}}$$

Example n° 18 Two Unrelated Circles

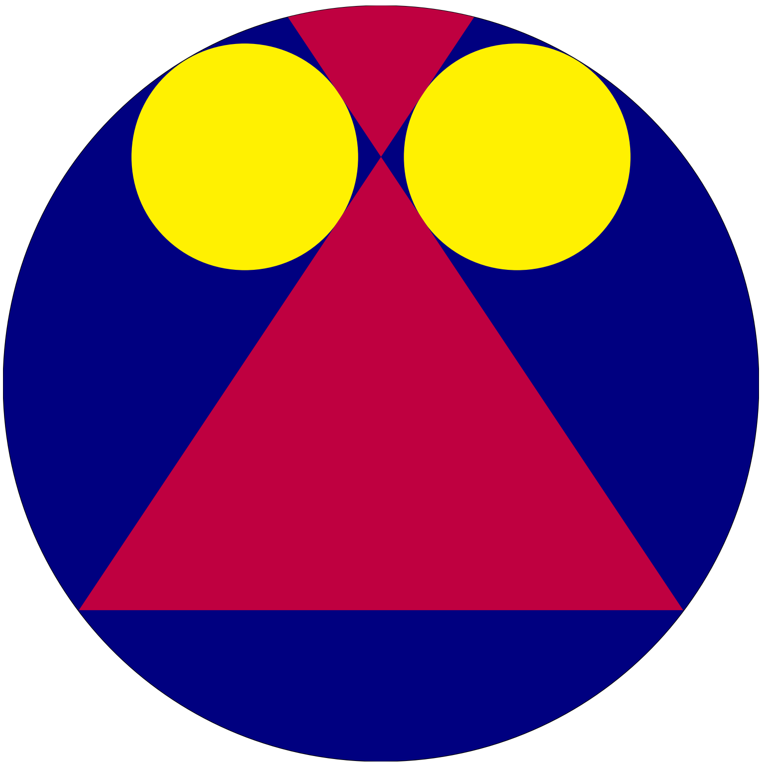
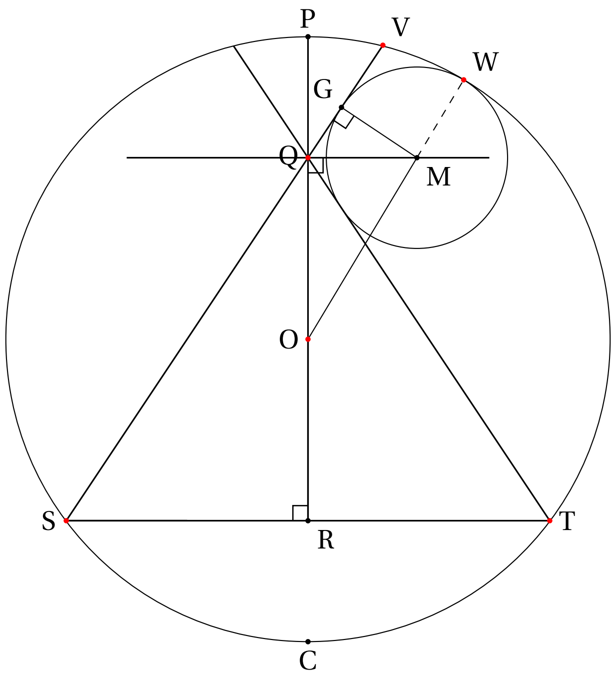


Figure 1: Sangaku Two Unrelated Circles

Now you can read the code to get the last picture.

```
\begin{tikzpicture}
    \tkzInit[xmin = -5,ymin = -5,xmax = 5,ymax = 5]
    \tkzPoint*(0,0){O}\tkzPoint*(-2,-3){A}
    \tkzPoint*(2,-3){B}\tkzPoint*(0,3){Q}
    \tkzCircleR(O,5 cm)
    \tkzInterLCR(A,B)(O,5 cm){S}{T}
    \tkzInterLCR(S,Q)(O,5 cm){V}{X}
    \tkzClipCircle(O,5 cm)
    \tkzLine[prefix = dm](S/Q)\tkzLine[prefix = dn](T/Q)
    \tkzMidPoint*(S,T){R}
    \tkzLineOrth(O,Q)(Q)
    \tkzMathLength(R,Q)\let\dRQ\tkzmathLen%
    \tkzMathLength(S,Q)\let\dSQ\tkzmathLen%
    \pgfmathparse{\dSQ/\dRQ*1.5}
    \tkzPoint*(\pgfmathresult,3){M}\tkzPoint*(-\pgfmathresult,3){N}
    \tkzProjection*[pos = above left](V,Q)(M/G)
    \tkzInterLCR(O,M)(O,5 cm){W}{Y}
    \tkzLine(A/B,O/Q,S/Q,T/Q)
    \tkzCircleR(M,1.5 cm)
    \tkzFillCircle[color = blue!50!Black](O,5cm)
    \tkzFillCircle[color = yellow](M,1.5cm)
    \tkzFillCircle[color = yellow](N,1.5cm)
    \tkzFillPolygon[color = purple](S,T,Q)
    \tkzFillPolygon[color = purple](dmr,dnr,Q)
\end{tikzpicture}
```

Explanation :



Sangaku - Solution of Nathan Bowler

Take coordinates such that it is the unit circle ($r = 1$), with Q on the x axis.
Let $G = (a\,;\ b)$, $W = (u\,;\ v)$.
Then:

- The line (QV) has equation $ax + by = 1$,

- Q is at $\left(\frac{1}{a};0\right)$,
- O is at $\left(\frac{1}{a};\frac{v}{au}\right)$.
- V satisfies

$$ax+by=1 \text{ and } \left(x-\frac{1}{a}\right)^2+\left(y-\frac{v}{au}\right)^2=\left(1-\frac{1}{au}\right)^2.$$

Further

$$y^2-\frac{av}{u}2y+2\frac{a}{u}-a^2-1=0$$

so that

$$\begin{aligned}\mathrm{QR}&=-y\\&=-\frac{av}{u}-\sqrt{(\frac{a^2}{u^2}-2\frac{a}{u}+1)}\\&=-\frac{av}{u}+1-\frac{a}{u}\\&=1-\frac{a(1+v)}{u}.\end{aligned}$$

and

$$\mathrm{PQ}=1-\frac{1}{au}+\frac{v}{au}=1-\frac{1-v}{au}.$$

So

$$(\mathrm{PQ}-1)(\mathrm{QR}-1)=\frac{(1-v)(1+v)}{u^2}=1.$$

Equivalently,

$$\frac{1}{\mathrm{PQ}}+\frac{1}{\mathrm{QR}}=1=\frac{1}{r}.$$

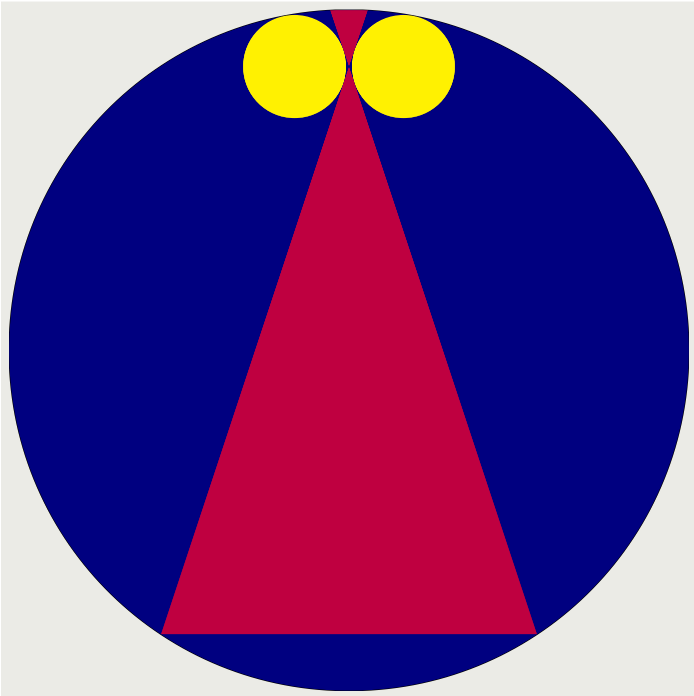
Figure 2: Sangaku-Two Unrelated Circles : R = 6 cm, OR = −2 cm et RQ = 4 cm

```
\begin{tikzpicture}[scale = .7]
   \TwoUnrelatedCircles{6}{-2}{4}
\end{tikzpicture}
```

In the general case, some information will be needed. For instance, it is necessary to give the radius of the big circle, the position of points R and Q. I have decided to give R$(0, r)$ relatively to O and Q relatively to P with the value of PQ.
The macro, below, is used to obtain some examples.

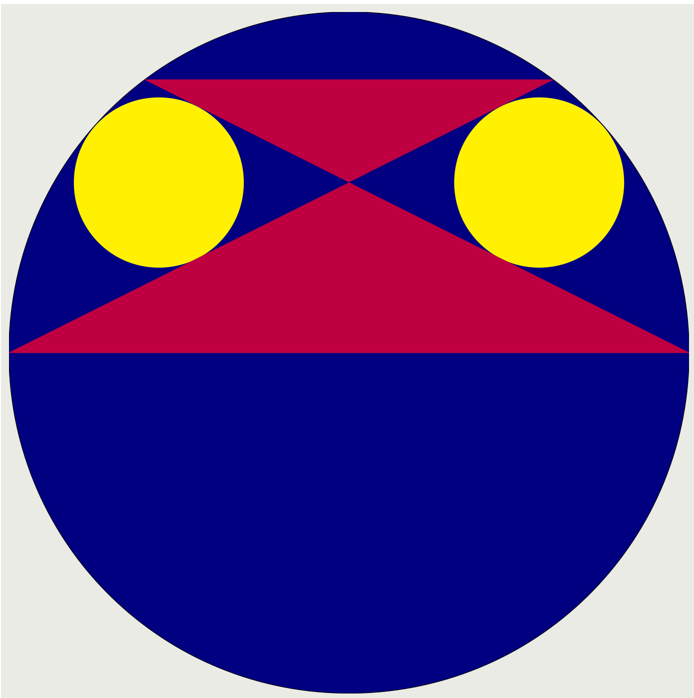
```
\newcommand*{\TwoUnrelatedCircles}[3]{
 % #1 --> R ; #2 --> Y_P <0 or >0 ; #3 --> PQ > =  0
\xdef\ORadius{#1}
\xdef\tucR{#2}
\xdef\tucQR{#3}
\pgfmathparse{\tucQR+\tucR}
\xdef\tucQ{\pgfmathresult}
\pgfmathparse{\ORadius-(\tucQR+\tucR)}
\xdef\tucPQ{\pgfmathresult}
\pgfmathparse{(\tucPQ*\tucQR)/(\tucPQ+\tucQR)}
\let\tucr\pgfmathresult%
\pgfmathparse{\ORadius-\tucr}
\let\OORadius\pgfmathresult%
\tkzInit[xmin = -\ORadius,ymin = -\ORadius,%
         xmax = \ORadius, ymax = \ORadius]\tkzClip
\tkzPoint(0,0){O}
\tkzPoint*(-\ORadius,\tucR){A}
\tkzPoint*(\ORadius,\tucR){B}
\tkzPoint(0,\tucQ){Q}
\tkzCircleR(O,\ORadius cm)
\tkzInterLCR(A,B)(O,\ORadius cm){S}{T}
\tkzInterLCR(S,Q)(O,\ORadius cm){S}{V}
\tkzInterLCR(Q,T)(O,\ORadius cm){Y}{T}
\tkzClipCircle(O,\ORadius cm)
\tkzLine[prefix = dm,kl = 3,kr = 5](Q/V)
\tkzLine[prefix = dn,kl = 3,kr = 5](Q/Y)
\tkzPoint(0,\tucR){R}
\tkzPoint(-\ORadius,\tucQ){U}
\tkzInterLCR(U,Q)(O,\OORadius cm){M}{N}
\tkzLine(S/Q,T/Q)
\tkzCircleR(M,\tucr cm)
\tkzFillCircle[color = blue!50!Black](O,\ORadius cm)
\tkzFillCircle[color = purple](M,\tucr cm)
\tkzFillCircle[color = purple](N,\tucr cm)
\tkzFillPolygon[color = orange](S,T,Q)
\tkzFillPolygon[color = orange](V,Y,Q)
}%
```

New examples



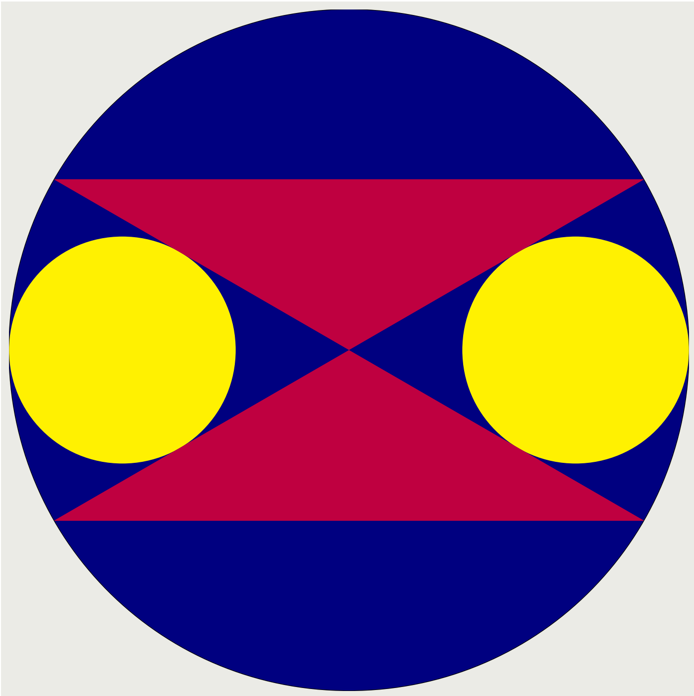
```
\begin{tikzpicture}[scale = .75]
   \TwoUnrelatedCircles{6}{-5}{10}
 \end{tikzpicture}
```

Figure 3: Sangaku-Two Unrelated Circles : R = 6 cm, OR = −5 cm et RQ = 10 cm



```
\begin{tikzpicture}[scale = .75]
   \TwoUnrelatedCircles{6}{0}{3}
 \end{tikzpicture}
```

Figure 4: Sangaku-Two Unrelated Circles : R = 6 cm, OR = 0 cm et RQ = 3 cm



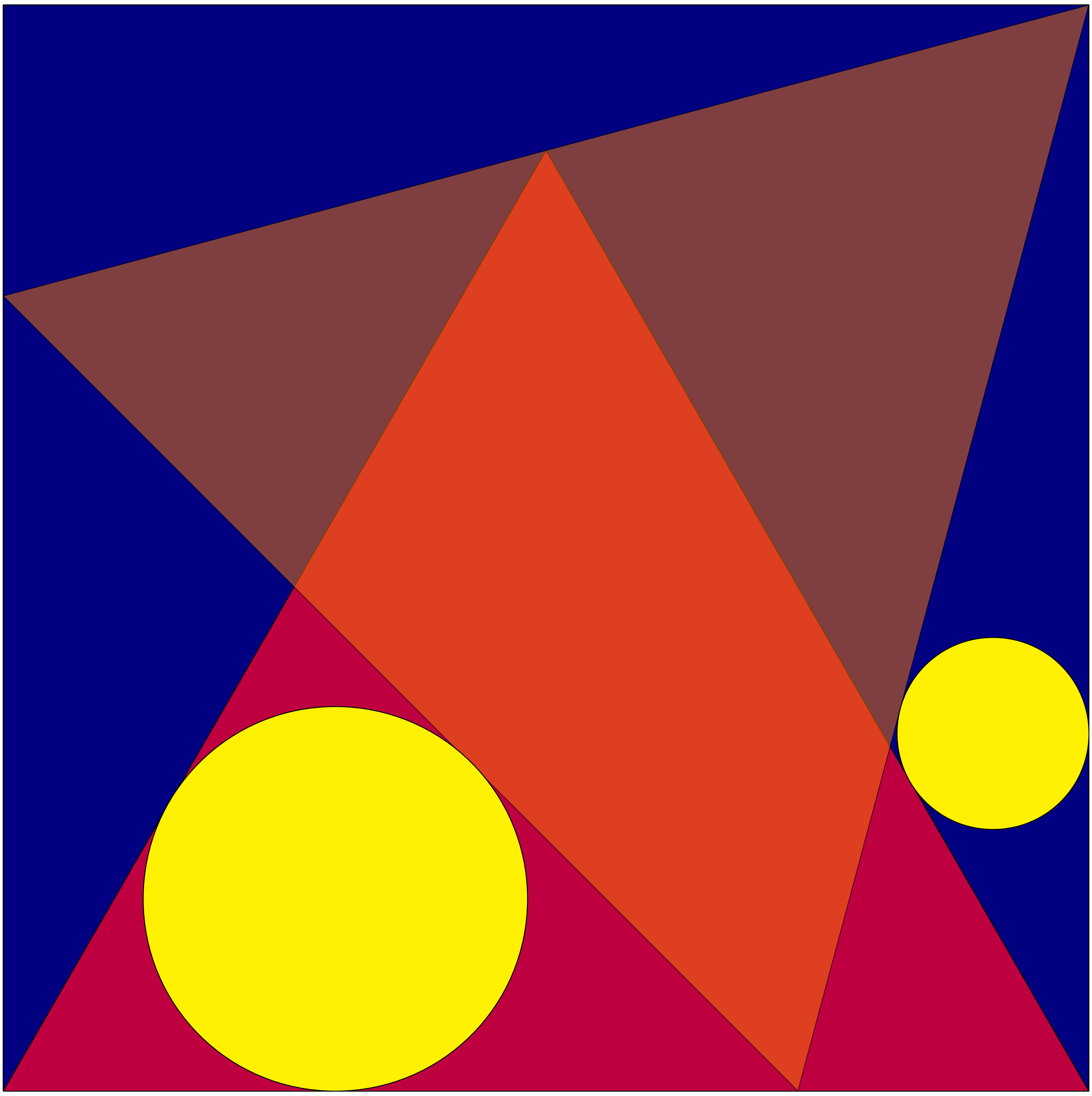
```
\begin{tikzpicture}[scale = .75]
   \TwoUnrelatedCircles{6}{-3}{3}
 \end{tikzpicture}
```

Figure 5: Sangaku-Two Unrelated Circles : R = 6 cm, OR = −3 cm et RQ = 3 cm

Sangaku in a square - I

Here is an elegant sangaku that requires both geometric and algebraic skills and some perseverance: Two equilateral triangles are inscribed into a square as shown in the diagram. Their side lines cut the square into a quadrilateral and a few triangles. Find a relationship between the radii of the two incircles shown in the diagram.

Example n° 19 Sangaku in a square - I



The code of the last figure is

```
\begin{tikzpicture}[scale = 1.75]
    \tkzInit
    \tkzPoint*(0,0){B} \tkzPoint*(8,0){C}%
    \tkzPoint*(0,8){A} \tkzPoint*(8,8){D}
    \tkzSquare*(B,C){D}{A}
    \tkzPolygon(B,C,D,A)\path[clip] (B)--(C)--(D)--(A)--cycle;
    \draw[fill = blue!50!black] (B)--(C)--(D)--(A)--cycle;
    \tkzTrEqui*(B,C){M}
    \draw[fill = red] (B)--(C)--(M)--cycle;
    \tkzInterLL*(D,M)(A,B){N}
    \tkzRotate*(N,-60)(D/L)
    \tkzBisector*(M,B,C){x}
    \tkzBisector*(N,L,B){y}
    \tkzInterLL*(L,y)(B,x){H}
    \tkzBisector*(M,C,D){u}
    \tkzBisector*(L,D,C){v}
    \tkzProjection*(C,B)(H/I)
    \draw[fill = orange,opacity = .5] (D)--(N)--(L)--cycle;
    \tkzCircle[style = {fill = yellow}](H,I)
    \tkzInterLL*(C,u)(D,v){K}
    \tkzProjection*(C,D)(K/J)
    \tkzProjection*(B,C)(M/E)
    \tkzCircle[style = {fill = yellow}](K,J)
\end{tikzpicture}
```

Firstly, we need to prove that it is possible that two equilateral triangles are inscribed into a square as shown in the diagram. A theorem exists but it is nice to find a solution in this particuler case. Let ABCD a square, BCM an equilateral triangle. The line (DM) intersects [AB] at point N. Then we construct a point L on the side [BC] and the angle $\widehat{\text{NDL}} = 60°$

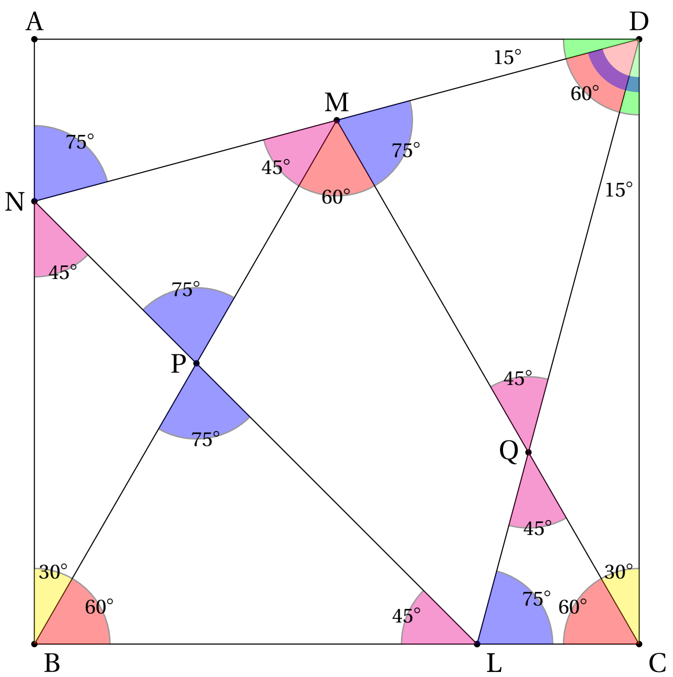
The triangle MCD is an isosceles triangle with two sides MC and CD of the same length a. It follows that

$\widehat{\text{MDC}} = \widehat{\text{DMC}} = 75°$ because $\widehat{\text{MCD}} = 30°$

Now we we can determine the angular size of all the angles

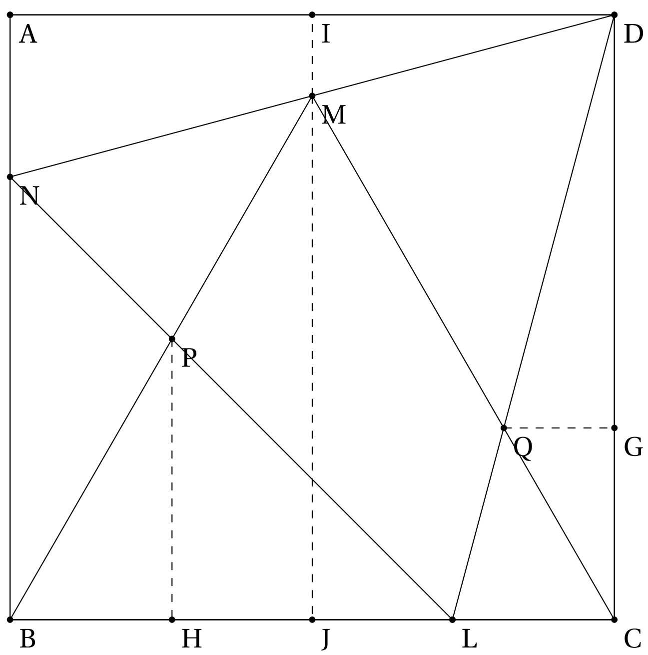
$\widehat{\text{LDC}} = 15°$ so $\widehat{\text{ADN}} = 15°$ and $\widehat{\text{AND}} = 75°$

AN = LC then BL = BN and $\widehat{\text{BLN}} = \widehat{\text{LNB}} = \widehat{\text{NMB}} = \widehat{\text{MQD}} = \widehat{\text{LQC}} = 45°$



```
\begin{tikzpicture}
    \tkzInit
    \tkzPoint(0,0){B}
    \tkzPoint(8,0){C}
    \tkzPoint*(0,8){A}
    \tkzPoint*(8,8){D}
    \tkzPolygon(B,C,D,A)
    \tkzTrEqui*(B,C){M}
    \tkzDrawPoint[pos = above](M,A,D)
    \tkzInterLL*(D,M)(A,B){N}
    \tkzRotate(N,-60)(D/L)
    \tkzDrawPoint(L)
    \tkzSegment(D/N,N/L,L/D,B/M,M/C)
    \tkzInterLL*(N,L)(M,B){P}
    \tkzInterLL*(M,C)(D,L){Q}
    \tkzDrawPoint[pos = left](N,P,Q)
    \tkzMarkAngle[fillcolor = green,dist = .40,label = 15^{\circ}](L/D/C)
    \tkzMarkAngle[fillcolor = green,dist = .35,label = 15^{\circ}](A/D/N)
    \tkzMarkAngle[fillcolor = yellow,dist = .20,label = 30^{\circ}](M/C/D,A/B/M)
    \tkzMarkAngle[fillcolor = magenta,dist = .20,label = 45^{\circ}]%
                 (B/L/N,L/N/B,N/M/B,M/Q/D,L/Q/C)
    \tkzMarkAngle[fillcolor = blue,dist = .20,label = 75^{\circ}]%
                 (A/N/D,N/P/M,B/P/L,C/L/D,C/M/D)
    \tkzMarkAngle[fillcolor = red,dist = .20,label = 60^{\circ}]%
                 (M/B/C,C/M/B,M/C/B,N/D/L)
    \tikzstyle{ai} = [draw,line width = .2cm]
    \tkzMarkAngle[size = .6,color = blue](C/D/M)
\end{tikzpicture}
```

We can see that the angles $\widehat{\mathrm{DNL}}$ and $\widehat{\mathrm{NLD}}$ have the same degree measurements 60°. DNL is an equilateral triangle and it is the largest equilateral triangle which can be inscribed in the square (Madachy 1979). We prove lately that the side is $s = (\sqrt{6} - \sqrt{2})a$.



```
\begin{tikzpicture}
    \tkzInit[xmin = -1,ymin = -1]\tkzClip
    \tkzPoint(0,0){B}
    \tkzPoint(8,0){C}
    \tkzPoint(0,8){A}
    \tkzPoint(8,8){D}
    \tkzSquare(B,C){D}{A}
    \tkzPolygon(B,C,D,A)
    \tkzTrEqui(B,C){M}
    \tkzInterLL(D,M)(A,B){N}
    \tkzRotate(N,-60)(D/L)
    \tkzDrawPoint(L)
    \tkzSegment(D/N,N/L,L/D)
    \tkzInterLL(N,L)(M,B){P}
    \tkzInterLL(M,C)(D,L){Q}
    \tkzProjection(B,C)(P/H)
    \tkzProjection(C,D)(Q/G)
    \tkzProjection(B,C)(M/J)
    \tkzProjection(D,A)(M/I)
    \tkzSegment[style = dashed](M/I,M/J,P/H,Q/G)
\end{tikzpicture}
```

We need some preliminaries to find the ratio between the radii of the two incircles shown in the first diagram.
Assume the side of the square equals a
First, we determine MI

$$\mathrm{MJ} = \frac{\sqrt{3}}{2}a \text{ and } \mathrm{MI} = a - \frac{\sqrt{3}}{2}a = \frac{(2-\sqrt{3})}{2}a$$

Thus we can find AN and NB

$$\mathrm{AN} = 2\mathrm{MI} = (2-\sqrt{3})a$$

and

$$\mathrm{BN} = \mathrm{AB} - \mathrm{AN} = a - (2-\sqrt{3})a = (\sqrt{3}-1)a$$

ADN is a right triangle with hypotenuse ND. We have, $\mathrm{AD}^2 + \mathrm{AN}^2 = \mathrm{ND}^2$ by the Pythagorean theorem. Using this, we continue:

$$\mathrm{ND}^2 = a^2 + (2-\sqrt{3})^2 a^2 = a^2(8-4\sqrt{3})$$

$$\mathrm{ND} = \mathrm{NL} = \mathrm{LD} = (\sqrt{6}-\sqrt{2})a$$

The value of $\tan(15^\circ)$ which will be useful later on.

$$\tan 15^\circ = \frac{\mathrm{AN}}{\mathrm{AD}} = \frac{(2-\sqrt{3})a}{a} = 2-\sqrt{3}$$

Now we can apply the standard formula in a triangle to determine the inradius

$$r = \frac{sh}{p}$$

where r, p, s, h are respectively the inradius, perimeter, a side and the altitude to the side in a triangle.
A good idea is to find a relationship between p and h
Let BPL the first triangle, here $h = \mathrm{PH}$, $l = \mathrm{BL}$ and $p = \mathrm{BP} + \mathrm{PL} + \mathrm{LB}$.

$$\sin(60^\circ) = \frac{\sqrt{3}}{2} = \frac{\mathrm{PH}}{\mathrm{BP}} = \frac{h}{\mathrm{BP}}$$

thus

$$\mathrm{BP} = \frac{2h}{\sqrt{3}}$$

HPL is an isosceles right triangle. The hypotenuse PL has length $\sqrt{2}h$.
And finally the relation between BL and h can be obtain like this

$$\mathrm{BL} = \mathrm{BH} + \mathrm{HL} = \frac{h}{\sqrt{3}} + h$$

In an other way

$$\mathrm{BL} = \mathrm{BN} = (\sqrt{3}-1)a$$

The inradius r_1 of BLP is

$$r_1 = \frac{sh}{p} = \frac{(\sqrt{3}-1)ah}{\dfrac{2h}{\sqrt{3}} + \sqrt{2}h + \dfrac{h}{\sqrt{3}} + h} = \frac{(\sqrt{3}-1)a}{1+\sqrt{2}+\sqrt{3}}$$

For CQD, similarly the inradius r_2 can be found with

$$r_2 = \frac{ah}{p}$$

with $p = \mathrm{CQ} + \mathrm{QD} + \mathrm{DC}$ and $h = \mathrm{QG}$
or

$$\frac{\mathrm{QD}}{\mathrm{DL}} = \frac{\mathrm{QG}}{\mathrm{LC}} = \frac{h}{(2-\sqrt{3})a}$$

from which

$$\mathrm{QD} = \frac{(\sqrt{6}-\sqrt{2})ah}{(2-\sqrt{3})a} = (\sqrt{6}+\sqrt{2})h$$

We continue

$$\frac{\mathrm{QG}}{\mathrm{QC}} = \frac{1}{2} = \frac{h}{\mathrm{GC}}$$

from which

$$\mathrm{QC} = 2h$$

and finally $\mathrm{DC} = \mathrm{DG} + \mathrm{GC} =$

$$\frac{h}{\mathrm{DG}} = \tan(15^\circ) = 2-\sqrt{3}$$

from which

$$\mathrm{DG} = \frac{h}{2-\sqrt{3}} = (2+\sqrt{3})h$$

and

$$\frac{\mathrm{GC}}{\mathrm{QC}} = \frac{\mathrm{GC}}{2h} = \frac{\sqrt{3}}{2}$$

Thus we can find

$$\mathrm{GC} = \sqrt{3}h$$

Finally

$$p = (\sqrt{6}+\sqrt{2})h + 2h + (2+\sqrt{3})h + \sqrt{3}h$$

The second radius is

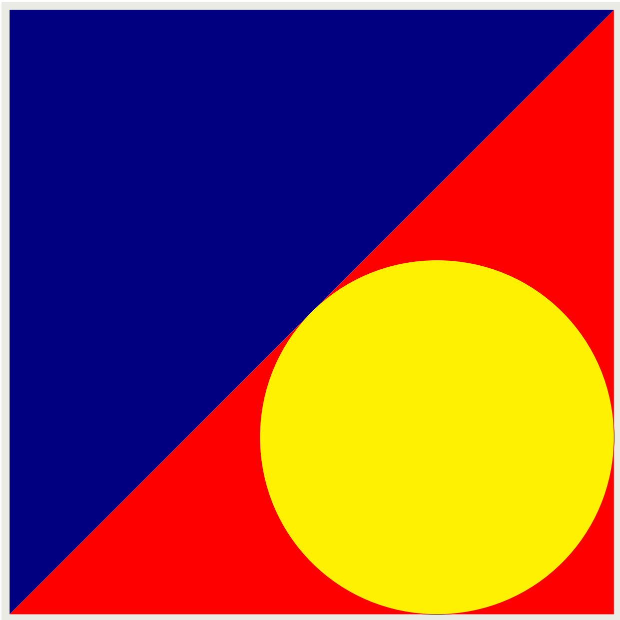
$$r_2 = \frac{ah}{p} = \frac{ah}{(\sqrt{6}+\sqrt{2})h + 2h + (2+\sqrt{3})h + \sqrt{3}h} = \frac{a}{4+\sqrt{2}+2\sqrt{3}+\sqrt{6}}$$

Without a special effort, we conclude that $r_1 = 2r_2$

Sangaku in a square - II

A very simple sangaku.
Find a relationship between the radius of the yellow circle and the side of the square.

Example n° 20 Sangaku in a square - II



```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint*(0,0){A} \tkzPoint*(8,0){B}
  \tkzSquare*(A,B){C}{D}
  \tkzProjection*(A,C)(B/F)
  \tkzBisector*(A,C,B){K}
  \tkzInterLL*(C,K)(B,F){I}
  \tkzCircle(I,F)
  \tkzMathLength(I,F)
  \tkzFillPolygon[color = blue!50!black](A,C,D)%
  \tkzFillPolygon[color = red](A,B,C)%
  \tkzFillCircle[color = yellow](I,\tkzmathLen pt)%
\end{tikzpicture}
```

Firstly, we can found the relationship between the inradius and the sides of a right triangle.
If r is the inradius of a circle inscribed in a right triangle with sides a and b and hypotenuse c, then

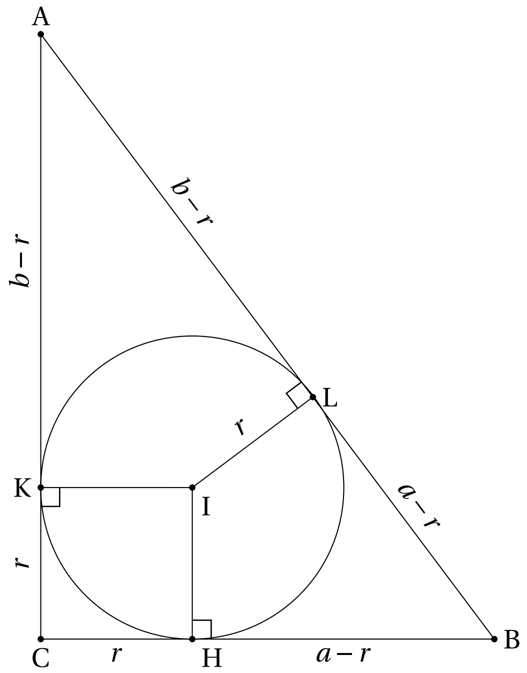
$$r = \frac{1}{2}(a+b-c).$$

Let ABC represents a right triangle, with the right angle located at C, as shown on the figure. Let a, b and c the lengths of the three sides; c is the length of the hypothenuse.
Let r and p be the radius of the incercle and the semiperimeter of the triangle.
a, b and c can be regarded in relation to r and they may be expressed with r: $a = r+(a-r)$, $b = r+(b-r)$ and $c = (a-r)+(b-r)$.
In a right triangle, we have the relation $r = p/2-c$. From the diagram, the hypotenuse AB is split in two pieces: $(a-r)$ and $(b-r)$, the length of the hypothenuse is $c = (a-r)+(b-r)$.
The perimeter is a function of r

$$p = a+b+c = r+(a-r)+r+(b-r)+(a-r)+(b-r) = 2a+2b-2r$$

so we can expressed r with s and c

$$2r = a + b - c = p - 2c \text{ and } r = \frac{p}{2} - c = \frac{a+b-c}{2}$$

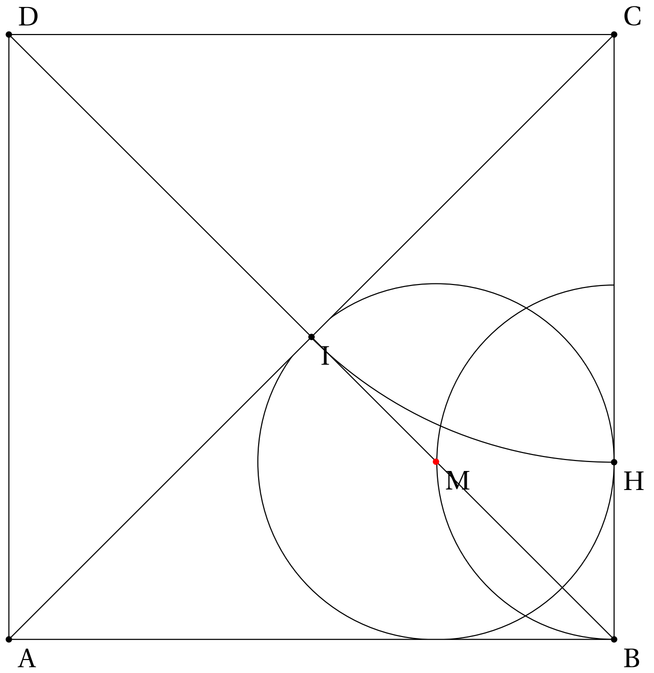



```
\begin{tikzpicture}
     \tkzInit\tkzClip[space = 0.5]
     \tkzPoint[pos = above](0,8){A}
     \tkzPoint[pos = right](6,0){B}
     \tkzPoint[pos = below](0,0){C}
     \tkzPolygon(A,B,C)%%
     \tkzInCenter(A,B,C){I}
     \tkzProjection(B,C)(I/H)
     \tkzProjection[pos = left](A,C)(I/K)
     \tkzProjection[pos = right](A,B)(I/L)
     \tkzSegment(I/L,I/H,I/K)
     \tkzCircle(I,H)
     \tkzRightAngle(A/L/I)
     \tkzRightAngle(B/H/I)
     \tkzRightAngle(C/K/I)
     \tkzSegmentMark[label = $r$,poslabel = -12pt](C/H)
     \tkzSegmentMark[label = $a-r$,poslabel = -12pt](H/B)
     \tkzSegmentMark[label = $r$](C/K)
     \tkzSegmentMark[label = $b-r$](K/A)
     \tkzSegmentMark[label = $a-r$](L/B)
     \tkzSegmentMark[label = $b-r$](A/L)
     \tkzSegmentMark[label = $r$](I/L)
\end{tikzpicture}
```

Now, let ABC represents a isosceles right triangle with AB = AC = a, then AC = $\sqrt{2}a$ and $a+b-c = 2a-\sqrt{2}a$
So the inradius in this case is

$$r = \frac{2-\sqrt{2}}{2}a$$

Now we can obtain the incenter without the bisectors

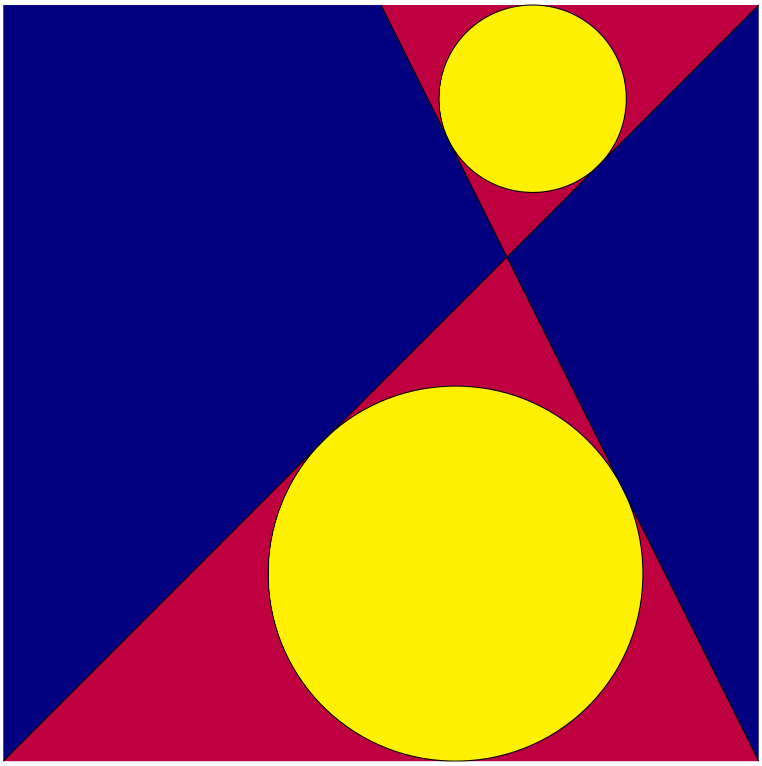



```
\begin{tikzpicture}[scale = 1.25]
  \tkzInit[xmin = -1,ymin = -1,xmax = 9,ymax = 9]
  \tkzClip
  \tkzPoint(0,0){A}
  \tkzPoint(8,0){B}
  \tkzPoint[pos = above right](8,8){C}
  \tkzPoint[pos = above right](0,8){D}
  \tkzSegment(A/B,B/C,C/D,D/A,A/C,B/D)
  \tkzInterLL(A,C)(B,D){I}
  \tkzDuplicateLength(C,I)(C,B){H}
  \tkzClipPolygon(A,B,C)
  \tkzCircle(C,I)
  \tkzCircle(H,B)
  \tkzMathLength(H,B)
  \tkzInterLCR(B,D)%
   (H,\tkzmathLen pt){M}{N}
  \tkzDrawPoint[color = red](M)
  \tkzCircle(M,H)
\end{tikzpicture}
```

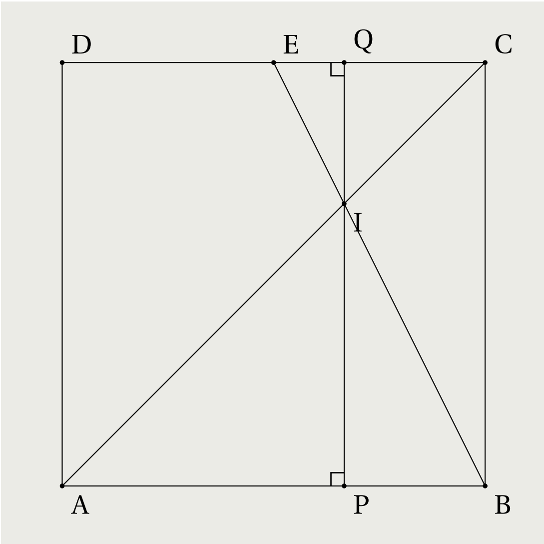
Sangaku in a square - III

In the following diagram, a triangle is formed by a line that joins the base of a square with the midpoint of the opposite side and a diagonal. Find the radius of the two inscribed circles.

Example n° 21 Sangaku in a square - III



```
\begin{tikzpicture}[scale = 1.25]
   \tkzInit\tkzClip
   \tkzPoint*(0,0){A}\tkzPoint*(8,0){B}
   \tkzPoint*(4,8){E}\tkzSquare*(A,B){C}{D}
   \tkzBisector*(C,A,B){I}\tkzBisector*(A,B,E){J}
   \tkzInterLL(A,I)(B,J){K}\tkzProjection*(A,B)(K/H)
   \tkzBisector*(D,C,A){I1}\tkzBisector*(C,E,B){J1}
   \tkzInterLL(C,I1)(E,J1){K1}\tkzProjection*(C,D)(K1/H1)
   \tkzFillPolygon[color = blue!50!black](A,B,C,D)
   \tkzFillPolygon[color = purple](A,C,E,B)
   \tkzCircle[style = {fill = yellow}](K,H)
   \tkzCircle[style = {fill = yellow}](K1,H1)
   \tkzSegment(E/B,A/C)
\end{tikzpicture}
```




```
\begin{tikzpicture}[scale = .7]
  \tkzInit[xmin = -1,ymin = -1,%
           xmax =  9,ymax =  9]
  \tkzClip
  \tkzPoint(0,0){A}
  \tkzPoint(8,0){B}
  \tkzPoint[pos = above right](4,8){E}
  \tkzPoint[pos = above right](8,8){C}
  \tkzPoint[pos = above right](0,8){D}
  \tkzPolygon(A,B,C,D)
  \tkzInterLL(A,C)(B,E){I}
  \tkzProjection(A,B)(I/P)
  \tkzProjection[pos = above right](C,D)(I/Q)
  \tkzRightAngle(A/P/I,D/Q/I)
  \tkzSegment(E/B,A/C,Q/P)
\end{tikzpicture}
```

QC is parallel to the base AB and is half as long which implies that the two triangles QIC and QAB are similar. I divides the segments QP and AD in ratio 2:1 so that

$$\mathrm{IP} = \frac{2}{3}\mathrm{QP} = \frac{2a}{3}$$

$$\mathrm{AI} = \frac{2}{3}\mathrm{AC}$$

Thus assuming AB = a, we have AC = $\sqrt{2}a$

$$\mathrm{AI} = \frac{2\sqrt{2}}{3}a$$

and

$$\mathrm{BI} = \frac{2}{3}\mathrm{BE}$$

we can apply the Pythagorean theorem to find BE

$$\mathrm{BE} = \frac{\sqrt{5}}{2}a$$

This means that

$$\mathrm{BI} = \frac{\sqrt{5}}{3}a$$

In any triangle,

$$r \times p = s \times h$$

where r is the inradius, p the perimeter, s the side and h the altitude of the triangle.
In other words

$$r\left(1 + \frac{\sqrt{5}}{3} + \frac{2\sqrt{2}}{3}\right)a = a \times \frac{2a}{3}$$

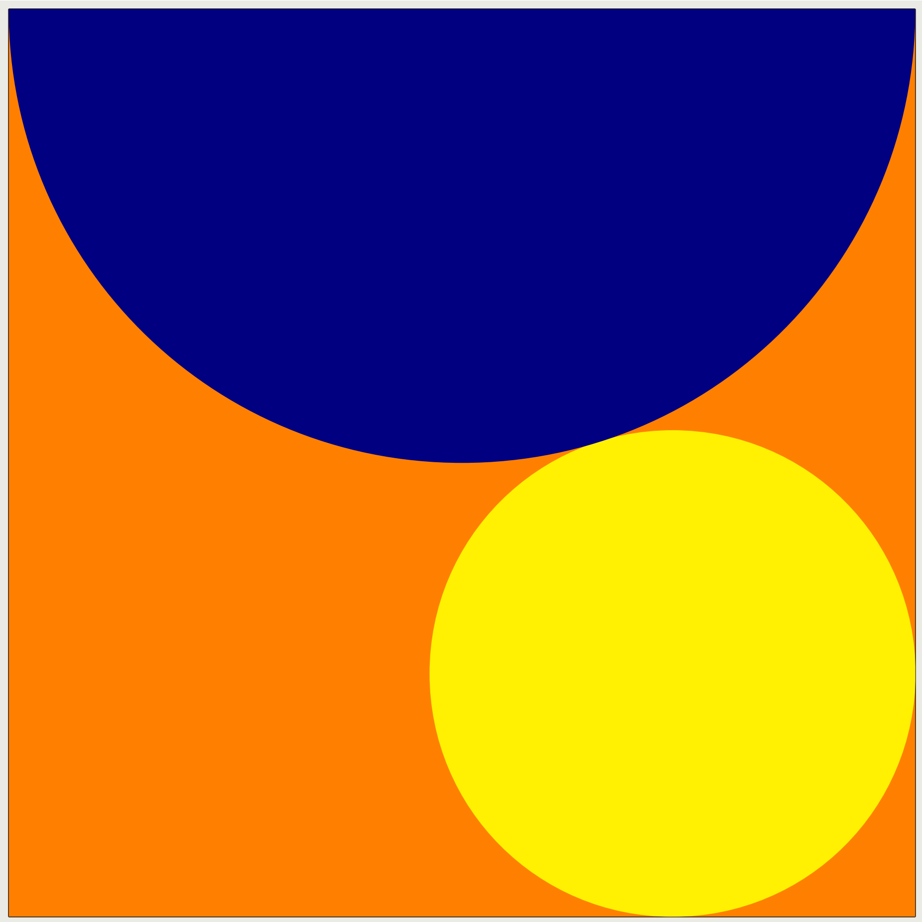
from which r is found:

$$r = \frac{2a}{3 + 2\sqrt{2} + \sqrt{5}}$$

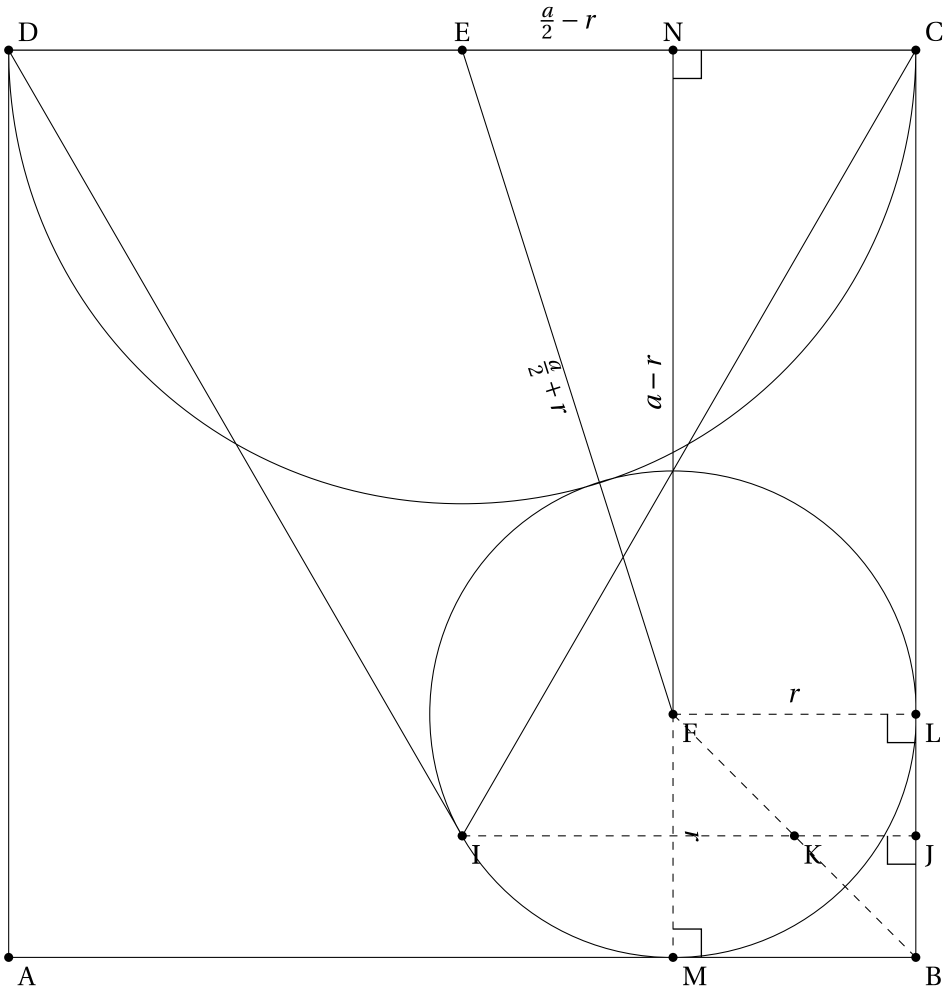
Sangaku in a square - IV

Find a relationship between the radius of the yellow circle and the side of the square.

Example n° 22 Sangaku in a square - IV



```
\begin{tikzpicture}[scale = 1.5]
   \tkzInit
   \tkzPoint*(0,0){A} \tkzPoint*(8,0){B}
   \tkzSquare*(A,B){C}{D}
   \tkzPolygon(B,C,D,A)
   \path[clip] (B)--(C)--(D)--(A)--cycle;
   \tkzPoint*(4,8){F}
   \tkzTrEqui(C,D){I}
   \tkzDrawPoint(I)
   \tkzProjection*(B,C)(I/J)
   \tkzInterLL*(D,B)(I,J){K}
   \tkzCSym*(K)(B/M)
   \tkzCircle(M,I)
   \tkzMathLength(M,I)
   \tkzFillPolygon[color = orange](A,B,C,D)
   \tkzFillCircle[color = yellow](M,\tkzmathLen pt)
   \tkzFillCircle[color = blue!50!black](F,4 cm)%
\end{tikzpicture}
```



Proof:
FNE is a right triangle with hypotenuse EF. We have, $EN^2 + NF^2 = EF^2$ by the Pythagorean theorem. In terms of a and r, the theorem appears as

$$\left(\frac{a}{2}-r\right)^2 + (a-r)^2 = \left(\frac{a}{2}+r\right)^2$$

which is equivalent to

$$4ar = a^2 + r^2$$

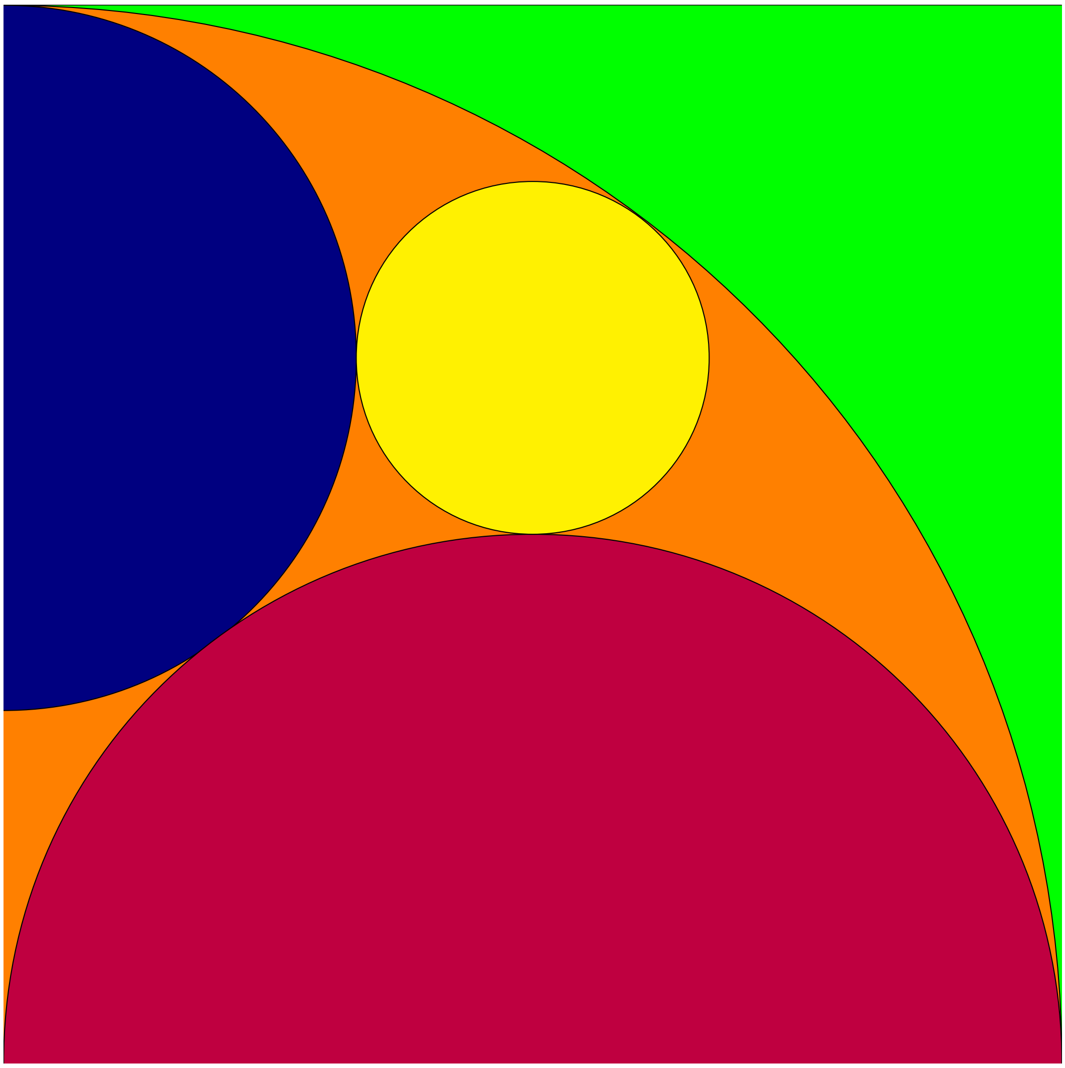
And finally

$$r = a(2-\sqrt{3})$$

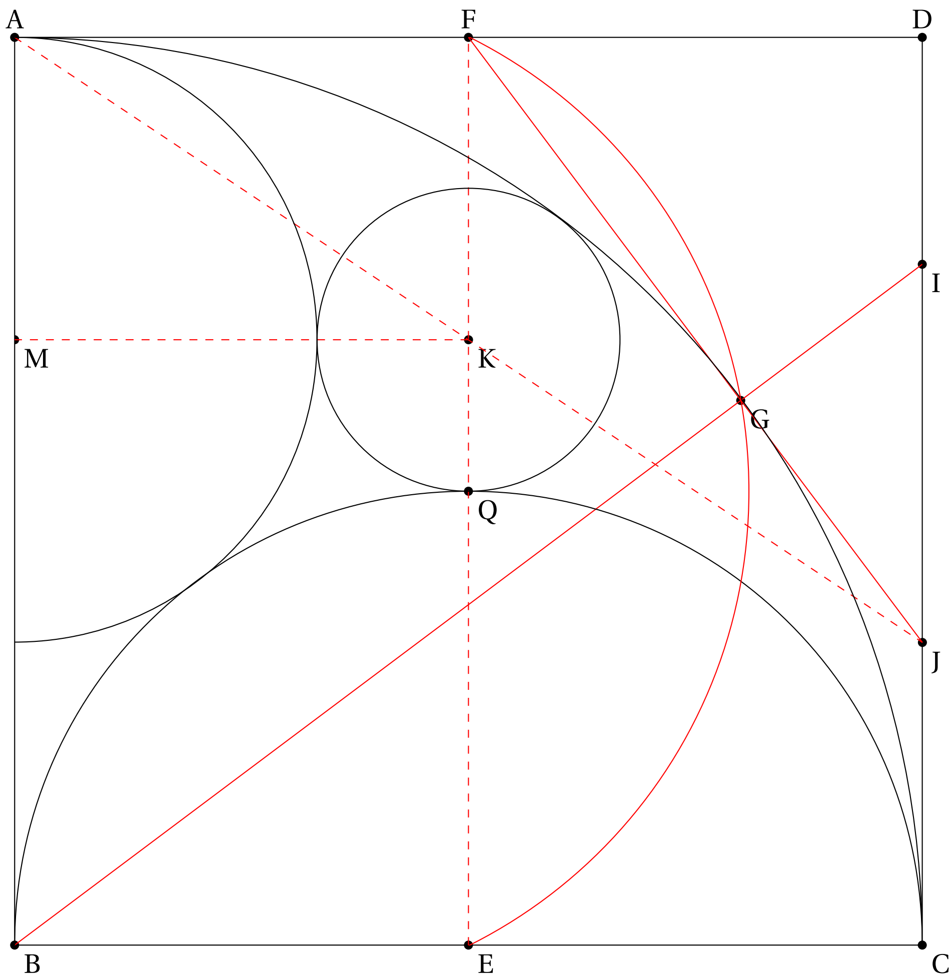
Sangaku in a square - V

This sangaku requires to determine the relative radii of the circles shown can be solved by an application of the Pythagorean theorem. Find a relationship between the radius of the circle and the side of the square.

Example n° 23 Sangaku in a square - V



```
\begin{tikzpicture}[scale = 1.75]
   \tkzInit[xmax = 8] \tkzClip
   \tkzPoint*(0,0){B} \tkzPoint*(8,0){C}
   \tkzPoint*(0,8){A} \tkzPoint*(8,8){D}
   \tkzPolygon(B,C,D,A)
   \path[clip] (B)--(C)--(D)--(A)--cycle;
   \tkzPoint*(4,8){F}\tkzPoint*(4,0){E}\tkzPoint*(4,4){Q}
   \tkzTgtFromP(F,F,A)(B){G}{H}
   \tkzInterLL*(F,G)(C,D){J}
   \tkzInterLL*(A,J)(F,E){K}
   \tkzProjection*(B,A)(K/M)
   \tkzFillPolygon[color = green](A,B,C,D)
   \tkzCircle[style = {fill = orange}](B,A)
   \tkzCircle[style = {fill = blue!50!black}](M,A)
   \tkzCircle[style = {fill = purple}](E,B)
   \tkzCircle[style = {fill = yellow}](K,Q)
\end{tikzpicture}
```



Assume the radius AM equal r and the side of the square is a.
step 1. Firstly, $\mathrm{ME}^2 = \mathrm{BE}^2 + \mathrm{BM}^2$ and the two circles are tangent if $\mathrm{ME} = r + \frac{a}{2}$. The equality becomes

$$\left(r+\frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + (a-r)^2$$

$$r = \frac{a}{3}$$

we have

$$\mathrm{ME} = \mathrm{BK} = \frac{5}{6}a$$

step 2.

$$\mathrm{KQ} = \mathrm{KE} - \mathrm{QE} = \frac{2a}{3} - \frac{a}{2} = \frac{a}{6}$$

and

$$\mathrm{MK} - \frac{a}{3} = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$$

The circle K is tangent mutually at the circle E and the circle M.

step 3. The little circle with center K is tangent at the circle B.

$$a - \frac{5a}{6} = \frac{a}{6}$$

step 4. A goog method to obtain K is finally to place I such as DI $= \frac{a}{4}$. Therefore, BI intercepts the big circle in G with GI $= \frac{a}{4}$,FG $= \frac{a}{2}$ and FG orthogonal to BG. FG intercepts BC in J such as DJ $= \frac{2a}{3}$. K is the common point between AJ and EF.

Sangaku - Circle Inscribing

Construct the figure consisting of a circle centered at O, *a second smaller circle centered at* $\mathrm{O_2}$ *tangent to the first, and an isosceles triangle whose base* $[\mathrm{AB}]$ *completes the diameter of the larger circle* $[\mathrm{XB}]$ *through the smaller* $[\mathrm{XA}]$. *Now inscribe a third circle with center* $\mathrm{O_3}$ *inside the large circle, outside the small one, and on the side of a leg of the triangle.*

Example n° 24 Circle Inscribing

References

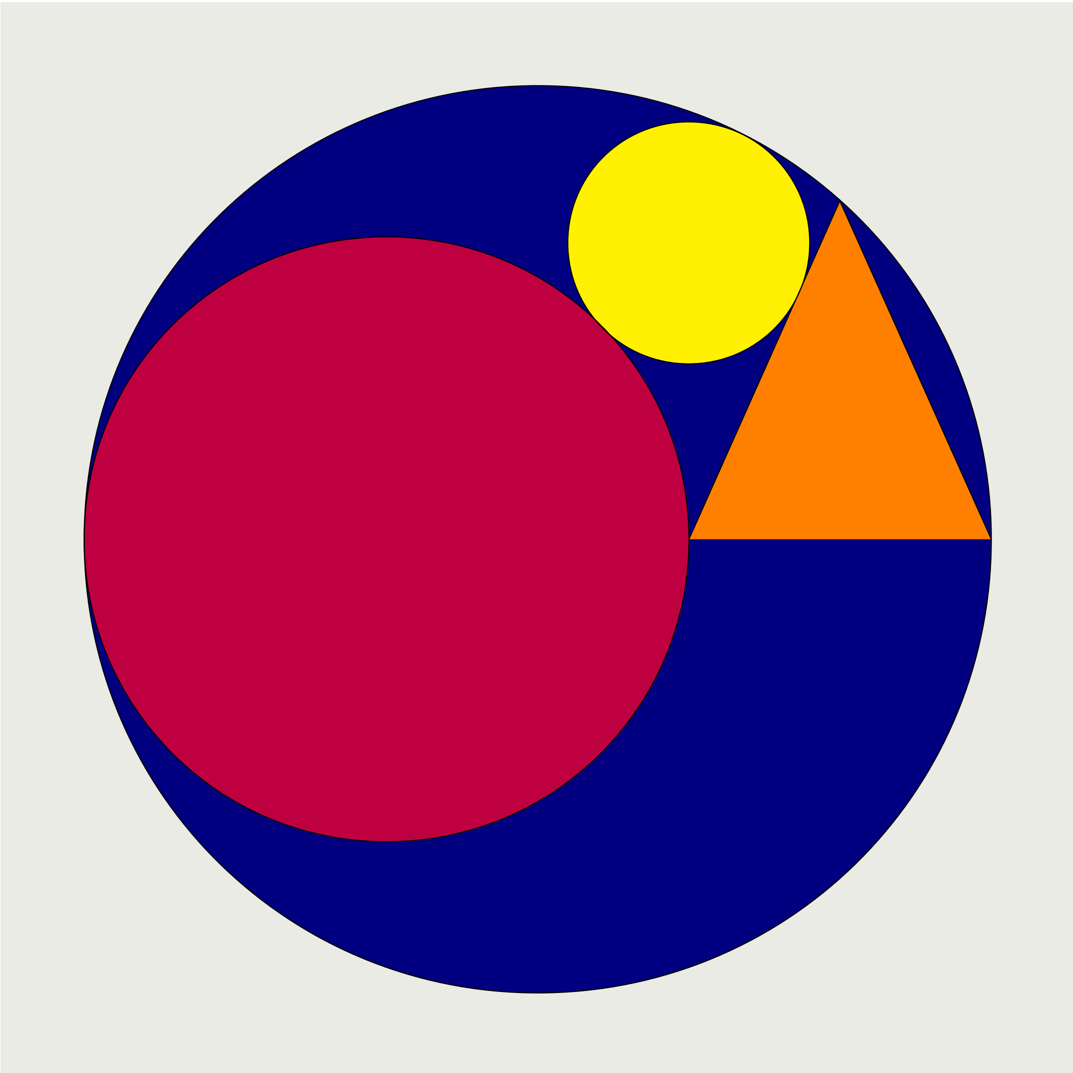
Weisstein, Eric W. "Circle Inscribing." From MathWorld–A Wolfram Web

http://mathworld.wolfram.com/CircleInscribing.html

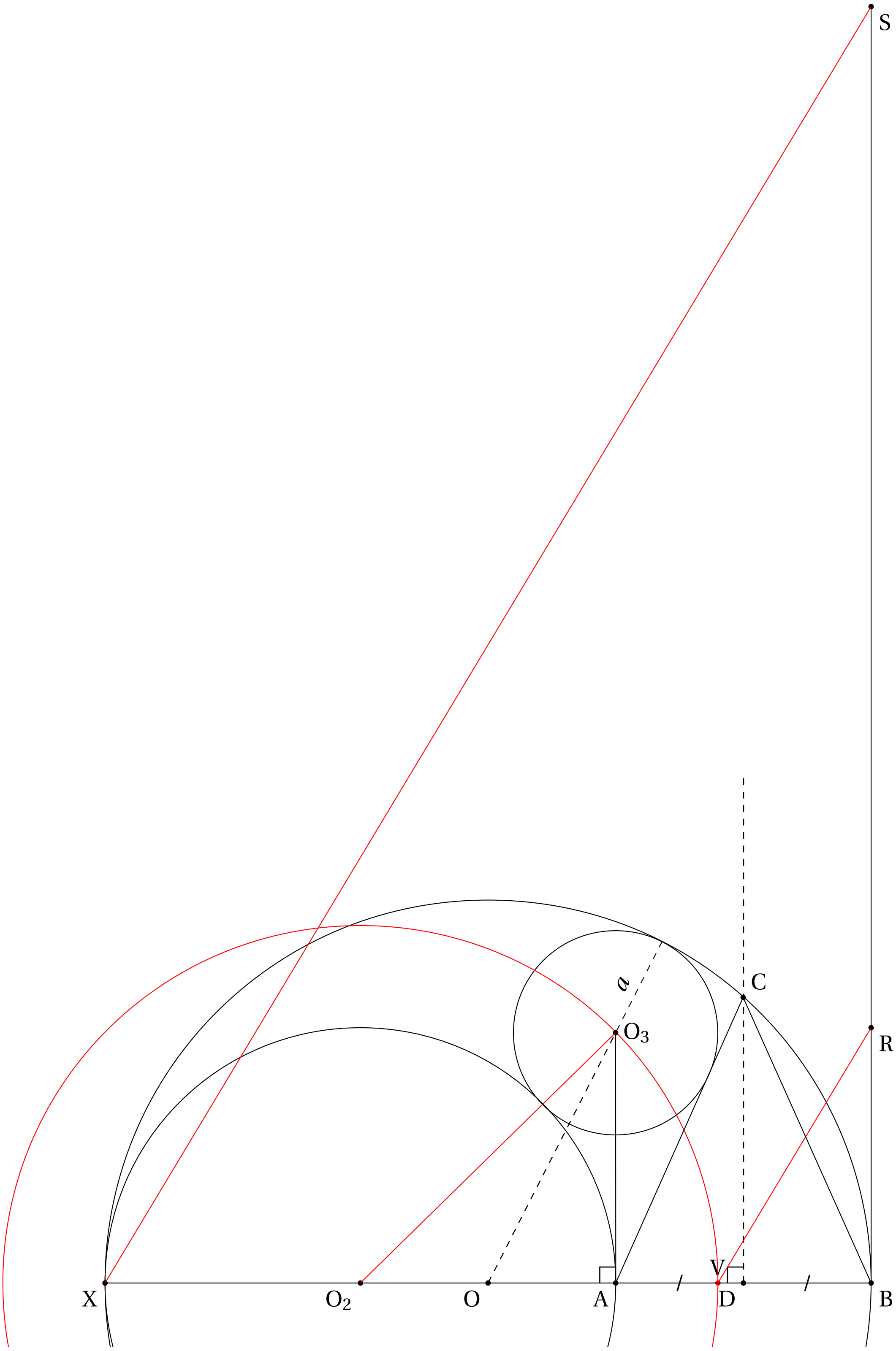
Alexander Bogomolny

http://www.cut-the-knot.org/

In this problem, from an 1803 sangaku found in Gumma Prefecture, the base of an isosceles triangle sits on a diameter of the large blue circle. This diameter also bisects the purple circle, which is inscribed so that it just touches the inside of the blue circle and one vertex of the orange triangle, as shown. The yellow circle is inscribed so that it touches the outsides of both the purple circle and the triangle, as well as the inside of the blue circle. A line segment connects the center of the yellow circle and the intersection point between the purple circle and the orange triangle. Show that this line segment is perpendicular to the drawn diameter of the blue circle.



```
\begin{tikzpicture}[scale = 1]
   \tkzInit[xmin = -7,xmax = 7,ymin = -7,ymax = 7]
   \tkzClip
   \xdef\ORadius{6}
   \xdef\OORadius{4}
   \pgfmathparse{(2*(\ORadius-\OORadius))/(\ORadius/\OORadius+1)}%
   \global\let\OOORadius\pgfmathresult%
   \pgfmathparse{\ORadius-\OOORadius}%
   \global\let\OOOORadius\pgfmathresult%
   \pgfmathparse{2*\OORadius-\ORadius}%
   \global\let\XA\pgfmathresult%
   \tkzPoint*(0,0){O}
   \ifdim\XA pt  =  0pt\relax%
      \tkzPoint[pos = below right](\XA,0){A}
   \else%
      \tkzPoint*(\XA,0){A}
   \fi%
   \tkzPoint*(\OORadius,0){D}
   \tkzPoint*(-\ORadius,0){X}
   \tkzPoint*(\ORadius,0){B}
   \tkzPoint*(\OORadius-\ORadius,0){O2}
   \tkzMediatorLine*[prefix = m](A,B)
   \tkzInterLCR(ml,mr)(O,\ORadius cm){C}{E}
   \tkzLineOrth*[prefix = p](X,A)(A)
   \ifdim\XA pt < 0 pt\relax%
      \tkzInterLCR(pl,pr)(O,\OOOORadius cm){O4}{O3}
   \else%
      \tkzInterLCR(pr,pl)(O,\OOOORadius cm){O3}{O4}
   \fi%
   \tkzInterLCR(O,O3)(O,\ORadius cm){W}{Z}
   \tkzFillCircle[color = blue!50!black](O,\ORadius cm)%
   \tkzFillPolygon[color = orange](A,B,C)%
   \tkzFillCircle[color = yellow](O3,\OOORadius cm)%
   \tkzFillCircle[color = purple](O2,\OORadius cm)%
   \tkzSegment(C/B,C/A)
   \tkzCircleR(O,\ORadius)
   \tkzCircleR(O2,\OORadius)
   \tkzCircleR(O3,\OOORadius)
\end{tikzpicture}
```


S
C
a
O_3
R
V
X
O_2
O
A
D
B

To find the explicit position and size of the circle, let the circle with center O have radius R and be centered at O and let the circle with center O_2 have radius r.

$$(r+a)^2 = r^2 + y^2$$

$$(\mathrm{R}-a)^2 = (\mathrm{R}-2r)^2 + y^2$$

for a and y gives

$$a = 2r\frac{\mathrm{R}-r}{\mathrm{R}+r}$$

$$y = \mathrm{AO}_3 = 2\sqrt{2\mathrm{R}}r\frac{\sqrt{\mathrm{R}-r}}{\mathrm{R}+r}$$

but now we need to prove that the circle is tangent to the line AC.
Let α the angle ACD and the angle $\mathrm{O}_3\mathrm{AC}$

$$\sin(\alpha) = \frac{\mathrm{AD}}{\mathrm{AC}}$$

$\mathrm{OD} = r$ and $\mathrm{AD} = \mathrm{R} - r$
OCD is a right triangle with hypotenuse OC = R. We have, $\mathrm{OD}^2 + \mathrm{CD}^2 = \mathrm{OC}^2$ by the Pythagorean theorem. In terms of r, the theorem appears as

$$r^2 + \mathrm{CD}^2 = \mathrm{R}^2$$

which is equivalent to

$$\mathrm{CD}^2 = \mathrm{R}^2 - r^2$$

and with the right triangle ADC and the Pythagorean theorem

$$\mathrm{AC}^2 = \mathrm{AD}^2 + \mathrm{CD}^2 = (\mathrm{R}-r)^2 + \mathrm{R}^2 - r^2 = 2\mathrm{R}(\mathrm{R}-r)$$

finally

$$\sin(\alpha) = \frac{\mathrm{AD}}{\mathrm{AC}} = \frac{\mathrm{R}-r}{\sqrt{2\mathrm{R}(\mathrm{R}-r)}} = \frac{\sqrt{\mathrm{R}-r}}{\sqrt{2\mathrm{R}}}$$

Let H the projection point of O_3 on the line AC, and d the length of $\mathrm{O}_3\mathrm{H}$

$$\sin(\alpha) = \frac{\mathrm{O}_3\mathrm{H}}{\mathrm{AO}_3} = \frac{d}{y} = \frac{d}{2\sqrt{2\mathrm{R}}r\dfrac{\sqrt{\mathrm{R}-r}}{\mathrm{R}+r}}$$

Using the two forms of sin(α)

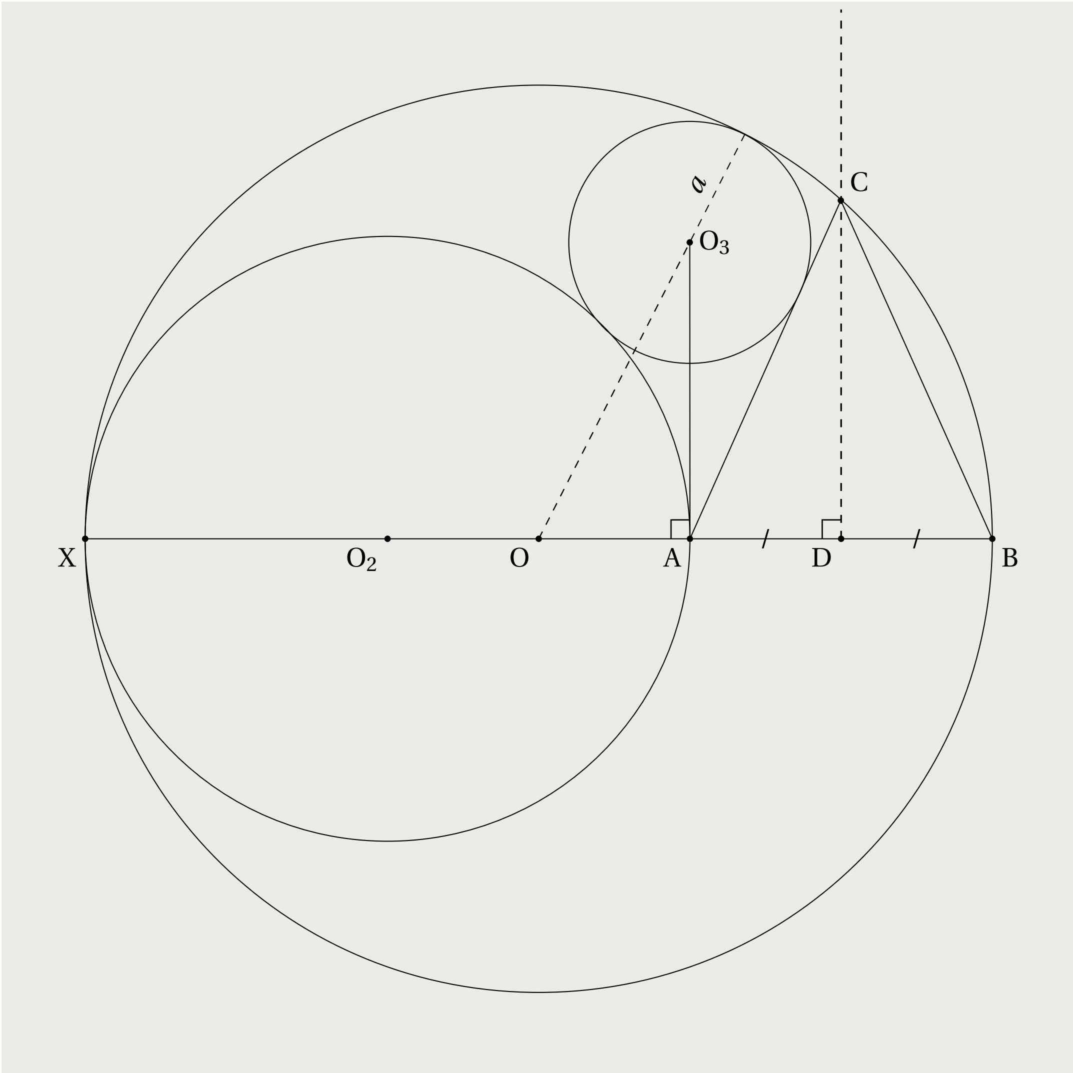
$$\frac{d}{2\sqrt{2\mathrm{R}}r\dfrac{\sqrt{\mathrm{R}-r}}{\mathrm{R}+r}} = \frac{\sqrt{\mathrm{R}-r}}{\sqrt{2\mathrm{R}}}$$

So

$$d = 2r\frac{\mathrm{R}-r}{\mathrm{R}+r} = a$$

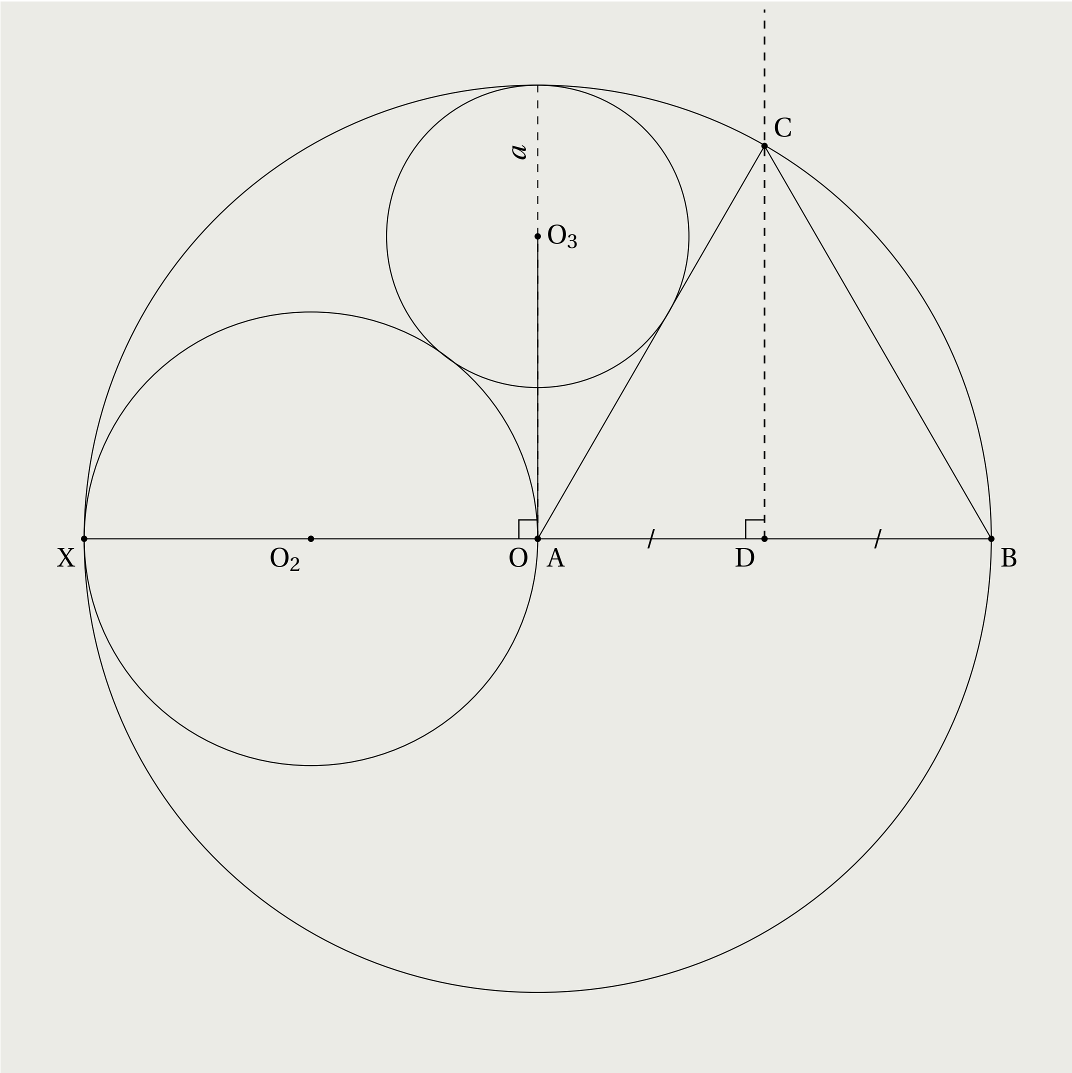
The code on the page is needed to get the examples.

```
\newcommand*{\CircleInscribing}[2]{%
    \xdef\ORadius{#1}
    \xdef\OORadius{#2}
    \pgfmathparse{(2*(\ORadius-\OORadius))/(\ORadius/\OORadius+1)}%
    \global\let\OOORadius\pgfmathresult%
    \pgfmathparse{\ORadius-\OOORadius}%
    \global\let\OOOORadius\pgfmathresult%
    \pgfmathparse{2*\OORadius-\ORadius}%
    \global\let\XA\pgfmathresult%
     \tkzPoint[pos = below left](0,0){O}
    \ifdim\XA pt  =  0pt\relax%
        \tkzPoint[pos = below right](\XA,0){A}
    \else%
        \tkzPoint[pos = below left](\XA,0){A}
    \fi%
    \tkzPoint[pos = below left](\OORadius,0){D}
    \tkzPoint[pos = below left](-\ORadius,0){X}
    \tkzPoint[pos = below right](\ORadius,0){B}
    \tkzPoint[name = $O_2$,pos = below left](\OORadius-\ORadius,0){O2}
    \tkzSegmentMark[symbol = /](D/B,D/A)
    \tkzCircleR(O,\ORadius)
    \tkzCircleR(O2,\OORadius)
    \tkzMediatorLine[prefix = m,kr = 2,kl = 0,style = dashed](A,B)
    \tkzInterLCR(ml,mr)(O,\ORadius cm){C}{E}
    \tkzLineOrth*[prefix = p](X,A)(A)
    \ifdim\XA pt < 0 pt\relax%
        \tkzInterLCR(pl,pr)(O,\OOOORadius cm){O4}{O3}
    \else%
        \tkzInterLCR(pr,pl)(O,\OOOORadius cm){O3}{O4}
    \fi%
   \tkzRightAngle(X/D/C,X/A/O3)
   \tkzCircleR(O3,\OOORadius)
   \tkzDrawPoint[name = $O_3$,pos = right](O3)
   \tkzDrawPoint[pos = above right](C)
   \tkzSegment[style = dashed](O/O3)
   \tkzSegment(A/O3,C/B,C/A,X/B)
   \tkzInterLCR(O,O3)(O,\ORadius cm){W}{Z}
   \tkzSegment[label = $a$,time = .85,style = dashed](O/Z)
}%
```



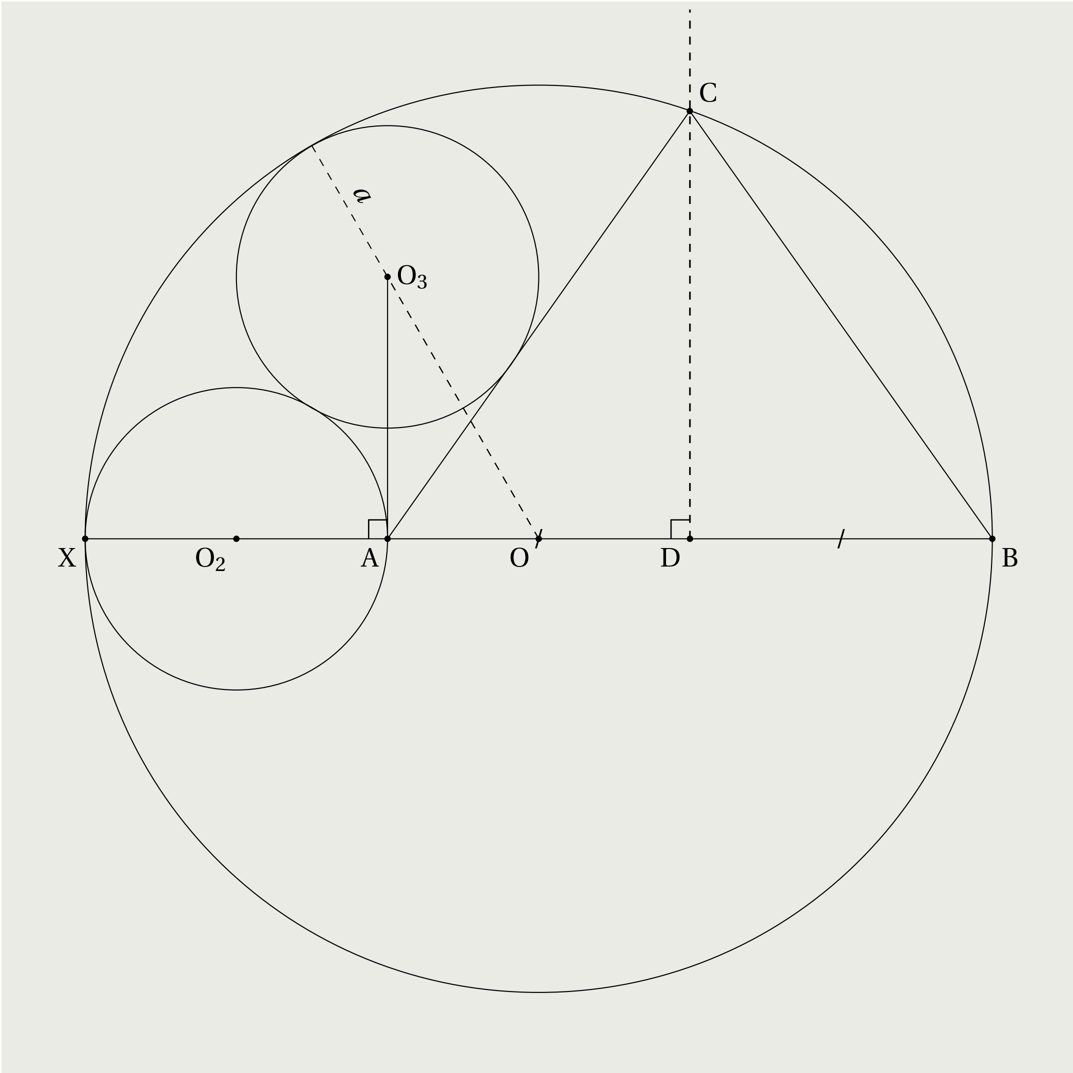

```
\begin{tikzpicture}[scale = 1]
   \tkzInit[xmin = -7,xmax = 7,ymin = -7,ymax = 7]
   \tkzClip
   \CircleInscribing{6}{4}
\end{tikzpicture}
```

Figure 6: Sangaku problem (1803) : R1 = 6 cm et R2 = 4 cm



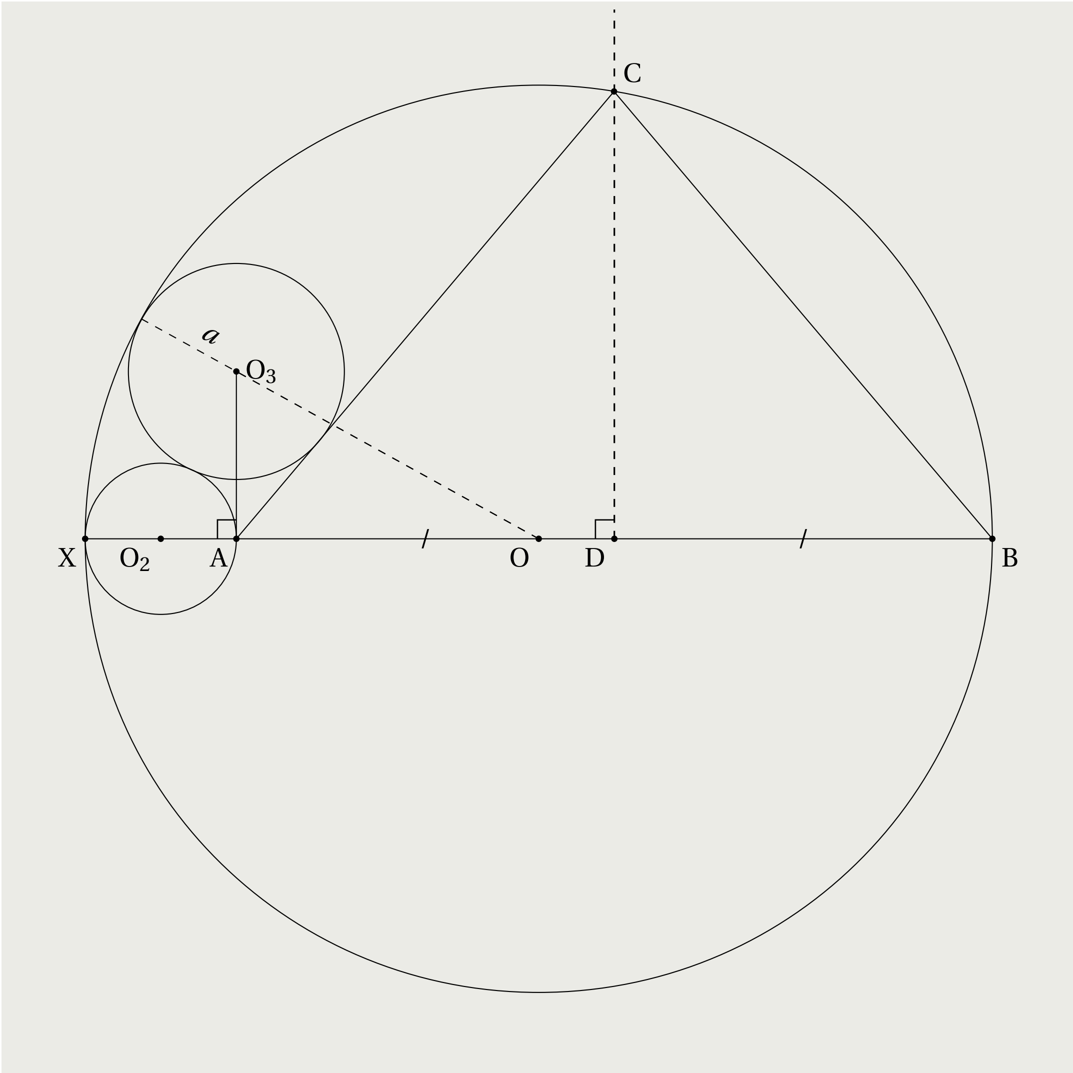

```
\begin{tikzpicture}[scale = 1]
   \tkzInit[xmin = -7,xmax = 7,ymin = -7,ymax = 7]
   \tkzClip
   \CircleInscribing{6}{3}
 \end{tikzpicture}
```

Figure 7: Sangaku problem (1803) : R1 = 6 cm et R2 = 3 cm




```
\begin{tikzpicture}[scale = 1]
   \tkzInit[xmin = -7,xmax = 7,ymin = -7,ymax = 7]
   \tkzClip
   \CircleInscribing{6}{2}
\end{tikzpicture}
```

Figure 8: Sangaku problem (1803) : R1 = 6 cm et R2 = 2 cm




```
\begin{tikzpicture}[scale = 1]
   \tkzInit[xmin = -7,xmax = 7,ymin = -7,ymax = 7]
   \tkzClip
   \CircleInscribing{6}{1}
\end{tikzpicture}
```

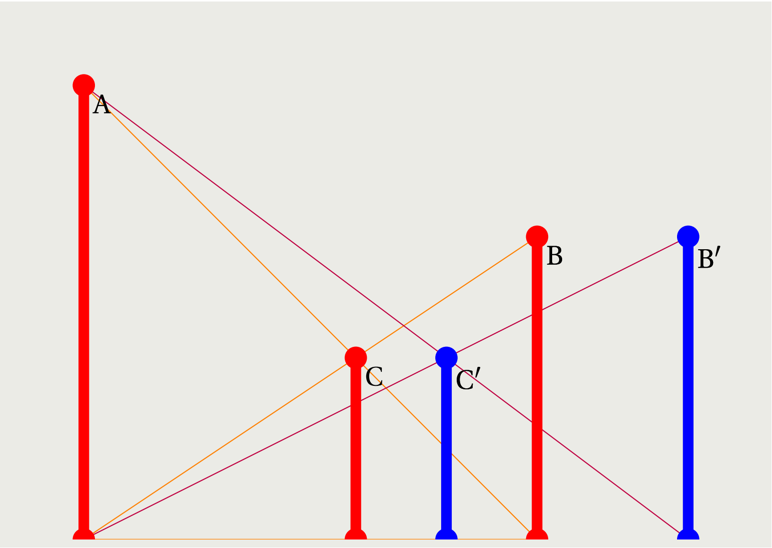
Figure 9: Sangaku problem (1803) : R1 = 6 cm et R2 = 1 cm

Sangaku - Harmonic mean

Two vertical segments AI, BJ (AI $= a$ *and* BJ $= b$*), the intersection* C *of the diagonals is at the height that depends solely on* AI *and* BJ*. In fact*

$$\frac{1}{\mathrm{CK}} = \frac{1}{\mathrm{AI}} + \frac{1}{\mathrm{BJ}} = \frac{1}{a} + \frac{1}{b}$$

Example n° 25 Harmonic mean




```
\begin{tikzpicture}
  \tkzInit[xmin = -1,xmax = 9,ymax = 7]\tkzClip
  \tkzPoint(0,6){A}\tkzPoint(0,0){I}
  \tkzPoint(6,4){B}\tkzPoint(8,4){B'}
  \tkzPoint(6,0){J}\tkzPoint(8,0){J'}
  \tkzInterLL*(A,J)(B,I){C}
  \tkzProjection(I,J)(C/K)
  \tkzInterLL(A,J')(B',I){C'}
  \tkzProjection(I,J)(C'/K')
  \tkzSegment[color = orange](A/J,B/I,I/J)
  \tkzSegment[color = purple](A/J',B'/I)
  \tkzSegment[lw = 4pt,color = red](C/K,A/I,B/J)
  \tkzSegment[lw = 4pt,color = blue](C'/K',B'/J')
  \tkzDrawPoint[color = red,size = 4pt](A,I,C,K,B,J)
  \tkzDrawPoint[color = blue,size = 4pt](C',K',B',J')
\end{tikzpicture}
```

Let's denote AI $= a$, BJ $= b$, CK $= c$, IK $= \alpha$ and KJ $= \beta$.
Then from similar triangles AIJ and IBJ we have the proportion

$$\frac{c}{a} = \frac{\beta}{\alpha+\beta} \text{ and } \frac{c}{b} = \frac{\alpha}{\alpha+\beta}$$

We can add the two equalities

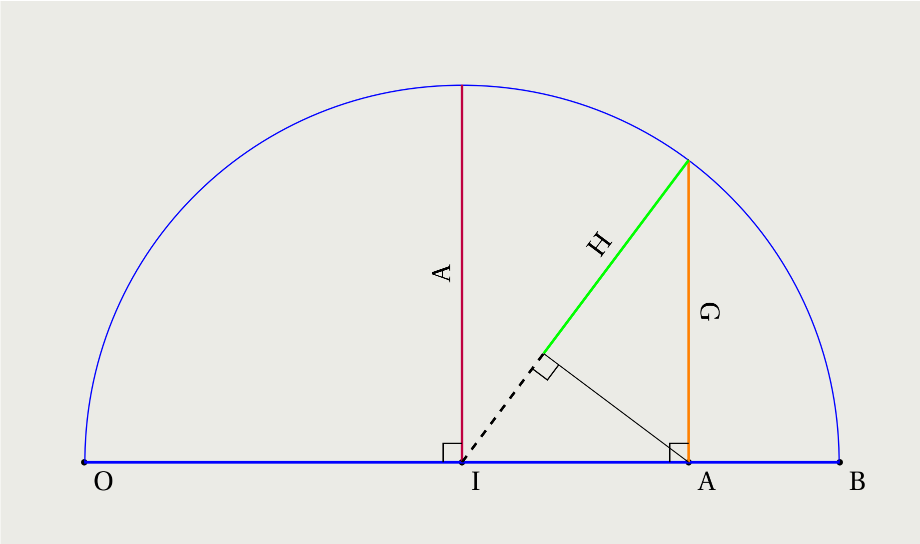
$$\frac{c}{a} + \frac{c}{b} = \frac{\beta}{\alpha+\beta} + \frac{\alpha}{\alpha+\beta} = 1$$

A division by c gives the desired result:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{c}$$

which says that c is the double of the harmonic mean of a and b.

Remark: a and b two numbers such as OA = a and AB = b




```
\begin{tikzpicture}
   \tkzInit[xmin = -1,ymin = -1,xmax = 11,ymax = 6]\tkzClip
   \tkzPoint(0,0){O}\tkzPoint(8,0){A}%
   \tkzPoint(10,0){B}\tkzPoint(5,0){I}
   \tkzPoint*(5,5){K}
   \tkzSegment[color = blue,lw = 1pt](O/B)
   \tkzClipSector(I,5 cm)(0,180)
   \tkzCircleR[color = blue,lw = 1pt](I,5 cm)
   \tkzLineOrth[prefix = h](O,B)(A)
   \tkzInterLCR(A,hr)(I,5 cm){G}{G'}
   \tkzSegment[lw = 1pt,color = purple](I/K)
   \tkzSegment[lw = 1pt,color = orange](A/G)
   \tkzProjection*(I,G)(A/H)
   \tkzSegment[lw = 1pt,color = green](H/G)
   \tkzSegment[lw = 1pt,style = dashed](I/H)
   \tkzRightAngle(O/I/K,O/A/G,A/H/I)
   \tkzSegment(A/H)
   \tkzSegmentMark[label = $A$](I/K)
   \tkzSegmentMark[label = $G$](A/G)
   \tkzSegmentMark[label = $H$](H/G)
\end{tikzpicture}
```

It is easy to find

$$\frac{\mathrm{H}}{\mathrm{G}} = \frac{\mathrm{G}}{\mathrm{A}}$$

In this case, we have $\mathrm{G}^2 = \mathrm{A} \times \mathrm{H}$ and $\mathrm{H} = \frac{\mathrm{G}^2}{\mathrm{A}}$
For two numbers a and b, $\mathrm{A} = \frac{a+b}{2}$ and $\mathrm{G} = \sqrt{ab}$, so

$$\frac{1}{\mathrm{H}} = \frac{\mathrm{A}}{\mathrm{G}^2} = \frac{a+b}{2ab}$$

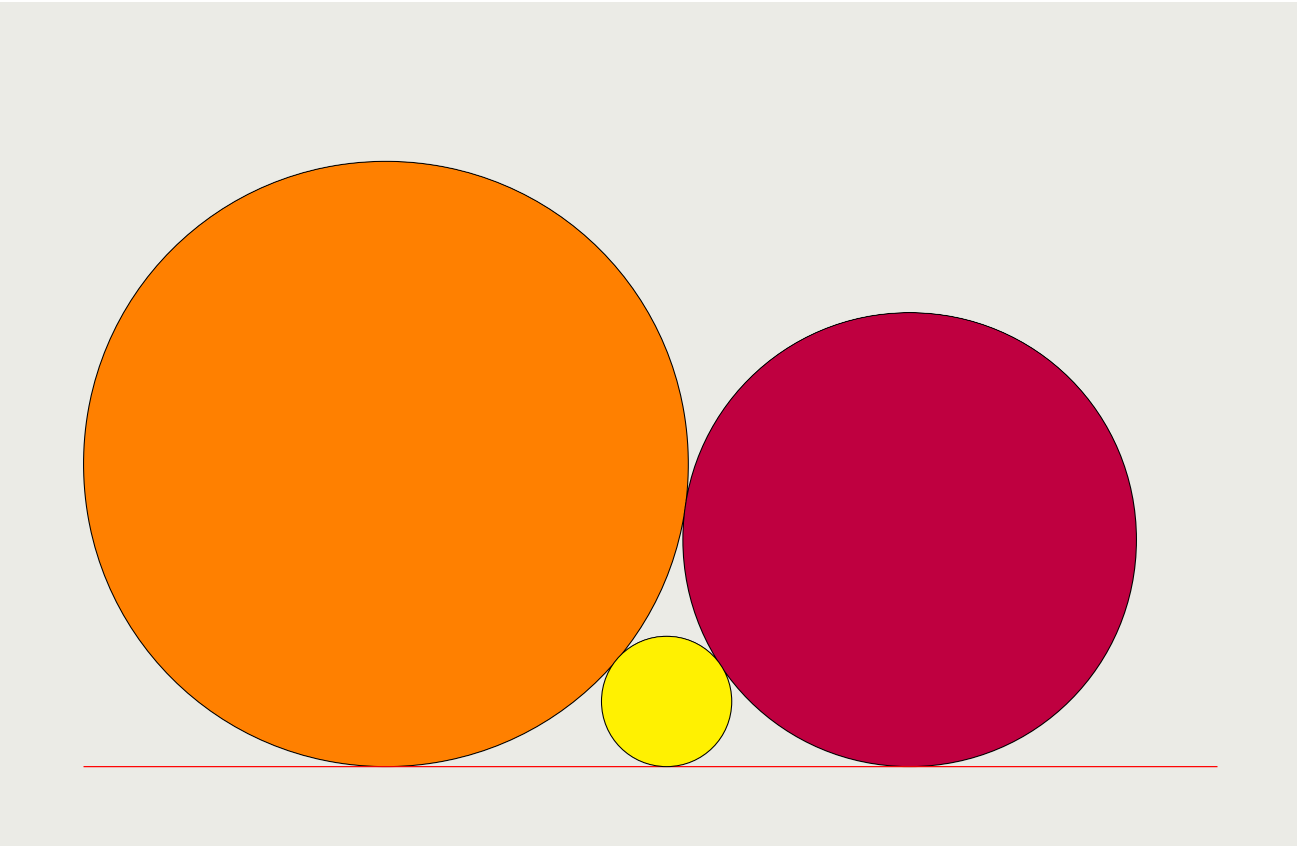
Finally

$$\frac{1}{\mathrm{H}} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

Sangaku - Three Tangent Circles

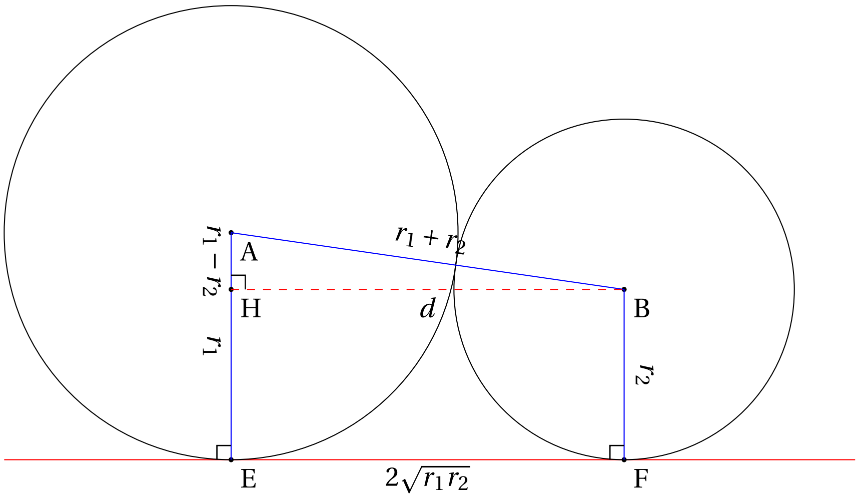
Given three circles tangent to each other and to a straight line, express the radius of the middle circle via the radii of the other two. This problem was given as a Japanese temple problem on a tablet from 1824 in the Gumma Prefecture (MathWorld)

Example n° 26 Three Tangent Circles



```
\begin{tikzpicture}
  \tkzInit[xmin = -1,ymin = -1,xmax = 16]\tkzClip
  \tkzPoint*(4,4){A}\tkzPoint*(10.928,3){B}
  \tkzPoint*(0,0){O}\tkzPoint*(15,0){X}
  \tkzSegment[color = red](O/X)
  \tkzCircleR[style = {fill = orange}](A,4 cm)
  \tkzCircleR[style = {fill = purple}](B,3 cm)
  \pgfmathparse{4+8*sqrt(3)/(2+sqrt(3))}
  \edef\cx{\pgfmathresult}
  \pgfmathparse{12/((2+sqrt(3))*(2+sqrt(3)))}
  \edef\cy{\pgfmathresult}
  \tkzPoint(\cx,\cy){C}
  \tkzSegment[color = red](O/X)
  \tkzCircleR[style = {fill = yellow}](C,\cy cm)
\end{tikzpicture}
```

As the diagram below shows we have three right triangles with the hypotenuses joining the centers of the three circles.
Using x and y to denote the horizontal distances between pairs of the circles, and R, R1, R2 at their radii, the triangles have the following sides:
$(r_1 - r)2 + x^2 = (r_1 + r)^2$ $(r_2 - r)^2 + y^2 = (r_2 + r)^2$ $(r_2 - r_1)2 + (x + y)2 = (r_2 + r_1)^2$. After simplification the equations become
$(x + y)2 = 4r_1r_2$

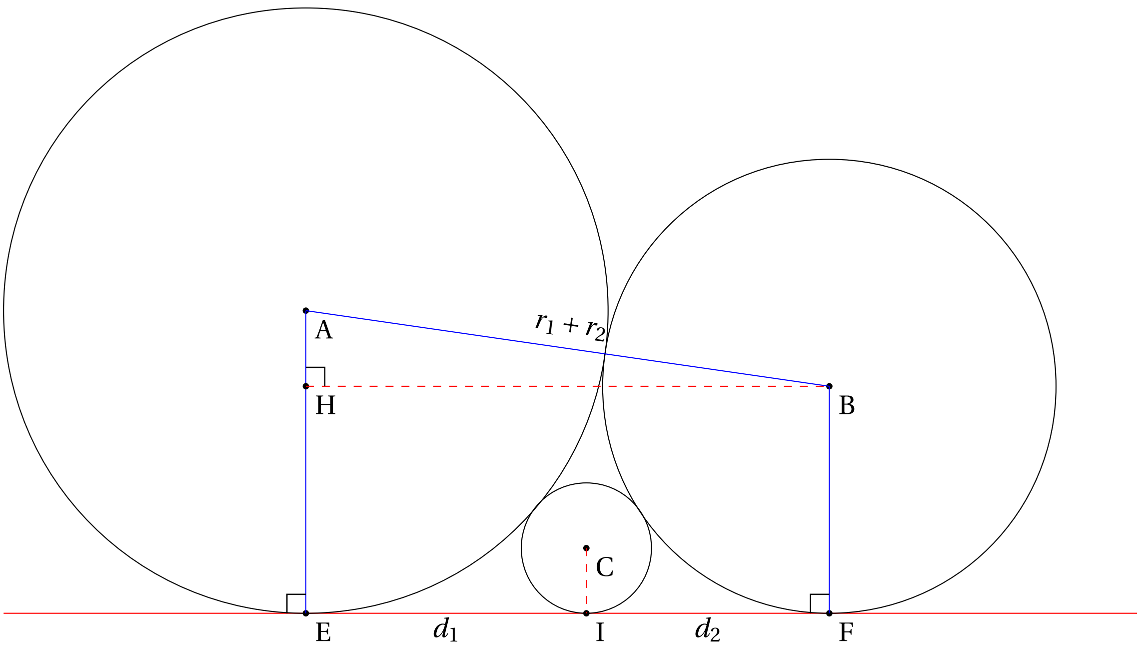


ABH is a right triangle with hypotenuse $AB = r_1 + r_2$. We have, $AH^2 + HB^2 = AB^2 = d^2$ by the Pythagorean theorem.
If two circles A and B and of radii r_1 and r_2 ($r_1 > r_2$) are mutually tangent to each other and a line OX, then their centers are separated by a horizontal distance given by solving

$$d^2 = (r_1 + r_2)^2 - (r_1 - r_2)^2$$

After simplification the equation gives

$$d = 2\sqrt{r_1 r_2}$$



Using d_1 and d_2 to denote the horizontal distances between pairs of the circles, and r, r_1, r_2 at their radii, the triangles have the following sides:

$$2\sqrt{r_1 r_2} = 2\sqrt{r_1 r} + 2\sqrt{r r_2}$$

Divide now by $\sqrt{r}\sqrt{r_1}\sqrt{r_2}$ to obtain

$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$$

$$\sqrt{r} = \frac{\sqrt{r_1}\sqrt{r_2}}{\sqrt{r_1}+\sqrt{r_2}}$$

$$d_1 = 2\sqrt{rr_1} = 2\sqrt{r_1}\frac{\sqrt{r_1}\sqrt{r_2}}{\sqrt{r_1}+\sqrt{r_2}} = \frac{2r_1\sqrt{r_2}}{\sqrt{r_1}+\sqrt{r_2}}$$

How to make this construction with a ruler and a compass

step 1. We want to draw two circles with centers A and B of radii $r_1 = 7cm$ and $r_2 = 4cm$ mutually tangent to each other and a line OX. OA $= 7cm$, OP $= 4cm$ and P$'$ is the symmetric point of p relatively the center O.

step 2. The circle with diameter AP$'$ intercepts the OX axis in a point I such as the length OI $= \sqrt{ab}$. It is easy to obtain H such OH $= \sqrt{ab}$. B is a point such as OHBP is a rectangle. We can draw the circle with center B and radius BH.

step 3. Now we can use the sangaku about harmonic mean. The line OB intercepts the line AH in a point K such as

$$\frac{1}{\mathrm{KJ}} = \frac{1}{\mathrm{OA}} + \frac{1}{\mathrm{HB}} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$$

$$\frac{1}{r} = \frac{1}{a} + \frac{1}{b} + \frac{2}{\sqrt{ab}}$$

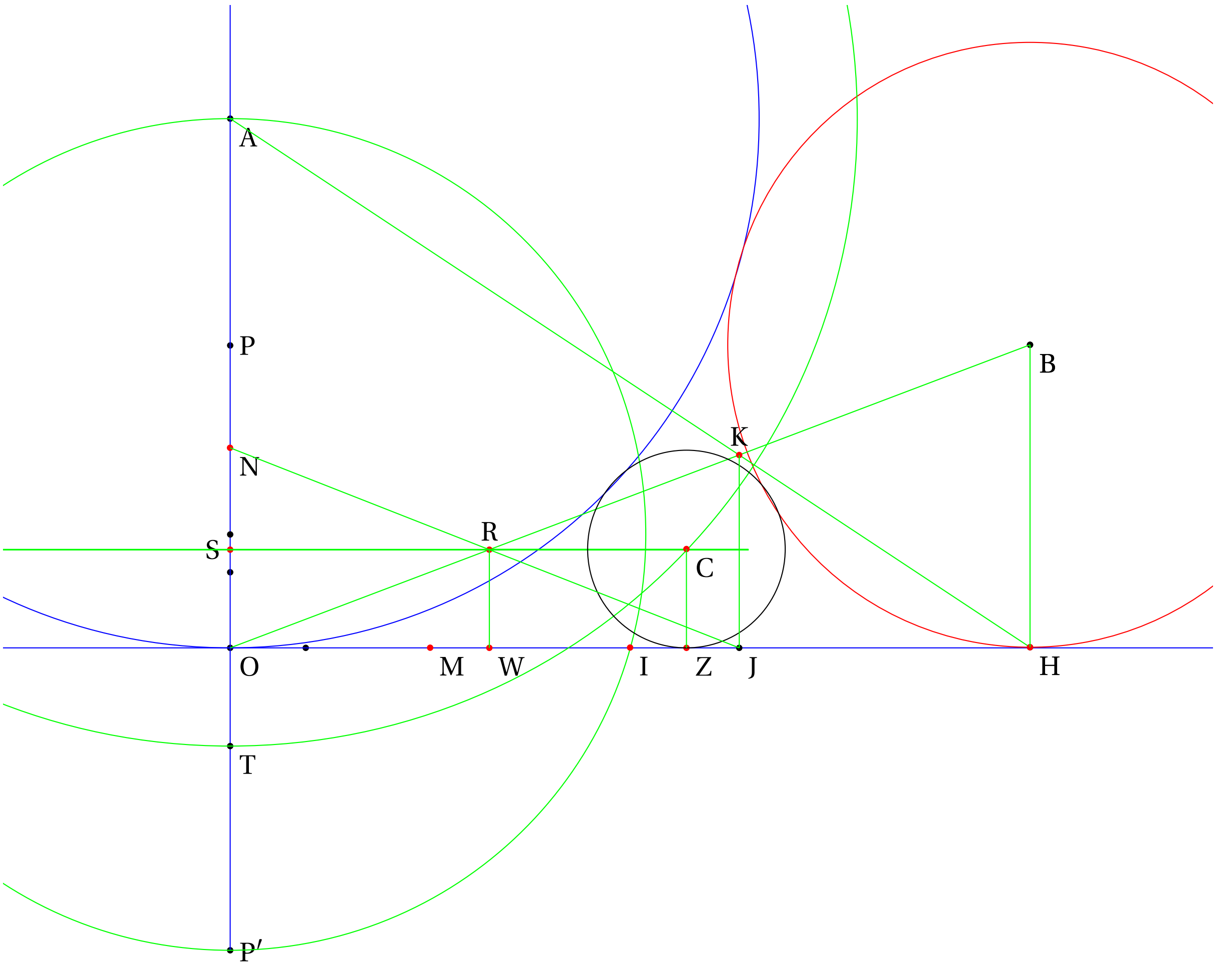
$$\frac{1}{r} = \frac{1}{\mathrm{KJ}} + \frac{1}{\frac{\sqrt{ab}}{2}}$$

To find r , we need only to represent $\frac{\sqrt{ab}}{2}$.

OI $= \sqrt{ab}$, OM $= \frac{\sqrt{ab}}{2}$ and ON = OM

step 4. The line NJ intercepts the line OK in a point R, if W is the projection of R on OX axis, we have RW $= r$. Let S the projection point of R on OY axis.

step 5. The point C is the intersection of the circle with center A and radius $a + r$ and the line (SR).




```
\begin{tikzpicture}
  \tkzInit[xmin = -3,ymin = -5,xmax = 13,ymax = 8.5]\tkzClip
  \tkzPoint(0,7){A}\tkzPoint(0,0){O}
  \tkzPoint[pos = right](0,4){P}\tkzPoint[pos = right](0,-4){P'}
  \tkzPoint[noname](0,1){V}\tkzPoint[noname](1,0){U}
  \tkzPoint*(16,0){X}\tkzPoint*(-5,0){X'}
  \tkzPoint*(0,16){Y}\tkzPoint*(0,-4){Y'}
  \tkzSegment[color = blue](X'/X,Y'/Y)
  \tkzCircleR[color = blue](A,7 cm)\tkzCircle*[color = green](A,P')
  \tkzMidPoint[noname](A,P'){pt1}
  \tkzInterLCR(O,X)(pt1,5.5 cm){BB}{I}\tkzCSym(I)(O/H)
  \tkzTranslation[color = red](O,H)(P/B)\tkzCircle[color = red](B,H)
  \tkzSegment[color = green](A/H,O/B)
  \tkzInterLL[color = red,pos = above](A,H)(O,B){K}
  \tkzProjection(O,X)(K/J)\tkzMidPoint[color = red](O,I){M}
  \tkzRotate[color = red](O,90)(M/N)
  \tkzInterLL[color = red,pos = above](N,J)(O,B){R}
  \tkzProjection[color = red,pos = left](O,A)(R/S)
  \tkzCSym(O)(S/T)\tkzCircle[color = green](A,T)
  \tkzLine[color = green](R/S)
  \tkzInterLCR(S,R)(A,\tkzmathLen pt){C'}{C}
  \tkzDrawPoint[color = red](C,I,H)\tkzProjection[color = red](O,X)(C/Z,R/W)
  \tkzCircle(C,Z)\tkzSegment[color = green](K/J,B/H,N/J,C/Z,R/W)
\end{tikzpicture}
```

VI . Basic Objects

For this documentation, I used the geometric French conventions for naming points, segments, lines. `tkz-2d` needs `tkz-base` and all environments `tikzpicture` with tkz-base needs `fp.sty`.
`tkzInit` is necessary and you can see, how to use it in the documentation of `tkz-base`. Often, I use the macro `\tkzClip`. When the first line is `\tkzInit[xmax=5,ymax=7]` then `\tkzClip` build a clip aera with the rectangle (0,0) and (5,7).

☞ Now this macro is more accurate because it is possible to use an option `space` like this : `\tkzClip[space=1]` and the last clip aera is the rctangle : (0-1,0-1) and (5+1,7+1).

macro n° 1 Define a point `\tkzPoint`

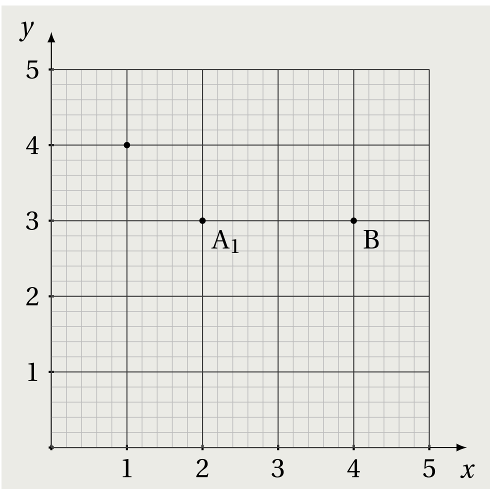
`\tkzPoint[⟨local options⟩](⟨xA ; yA⟩){⟨A⟩}`

options	default	definition
noname	`false`	si true pas de nom
name	`empty`	si non vide alors c'est le nom attribué au point
color	`black`	couleur du point
nomark	`false`	si true pas de marque
mark	`*`	représentation du point
namecolor	`black`	couleur du label
size	`\normalsize`	taille du point
pos	`above right`	position du nom
coord	`false`	booléen pour indiquer si on représente les coordonnées
xlabel	`empty`	nom de l'abscisse si coord = true
ylabel	`empty`	nom de l'ordonnée si coord = true
posxlabel	`0pt`	écart par rapport l'axe des abs. de xlabel
posylabel	`0pt`	écart par rapport l'axe des ord. de ylabel

This commands defines one geometrical point in the Euclidean plane, associated with a node (tikz object). Each point has a node name and two coordinates (⟨x ; y⟩) (unit=cm) and by default the node name defines the label (in text mode) put on the figure (examples : A, A1 *but not* A_1*). It is possible to assign a different label in mathematical mode like* A_1 *with an option* `name= $A_1$`

In the next example, the point A_1 has "A" for noe name but it has A_1 for label , C is defined but with no label and you can see its mark.

Example n° 27 Some points with different options




```
\begin{tikzpicture}
  \tkzInit[xmax=5,ymax=5]
  \tkzX
  \tkzY
  \tkzGrid[sub]
  \tkzPoint[name=$A_1$](2,3){A}
  \tkzPoint(4,3){B}
  \tkzPoint[noname](1,4){C}
\end{tikzpicture}
```

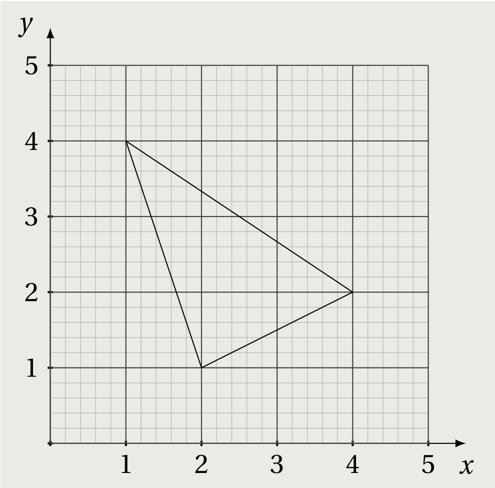
macro n° 2 Define a point without drawing it `\tkzPoint*`

`\tkzPoint*[⟨local options⟩](⟨xA ; yA⟩){⟨A⟩}`

*The *-form defines a point in the figure which is to be hidden.*

Example n° 28 The *-form of `\tkzPoint`

A, B and C are defined but with no name and no mark.



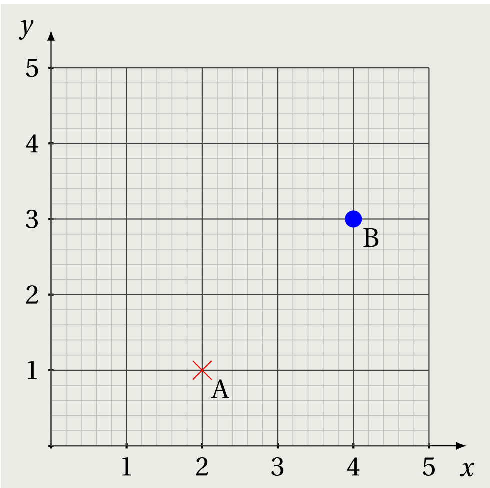

```
\begin{tikzpicture}
 \tkzInit[xmax=5,ymax=5]
 \tkzX
 \tkzY
 \tkzGrid[sub]
 \tkzPoint*(2,1){A}
 \tkzPoint*(4,2){B}
 \tkzPoint*(1,4){C}
 \tkzSegment(A/B,A/C,B/C)
%better is \tkzPolygob(A,B,C)
\end{tikzpicture}
```

macro n° 3 Drawing defined point(s) `\tkzDrawPoint`

`\tkzDrawPoint[⟨local options⟩](⟨A,B,...⟩)`

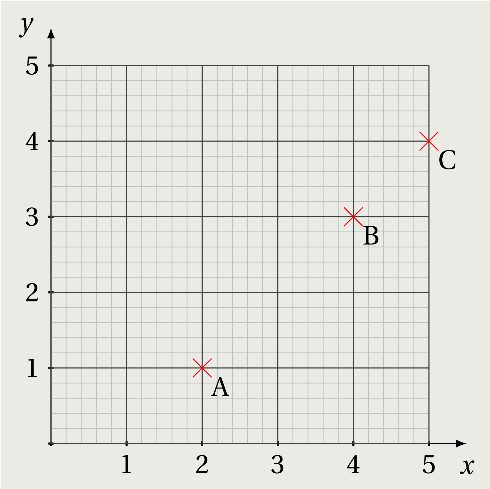
It is possible to define a point with drawing it immediatly. You can draw it later with this macro. You can use the same local options that for the point. You can draw the points of a list delimited by (. . .)

Example n° 29 Drawing a defined point




```
\begin{tikzpicture}
 \tkzInit[xmax=5,ymax=5]
 \tkzX
 \tkzY
 \tkzGrid[sub]
 \tkzPoint*(2,1){A}
 \tkzPoint*(4,3){B}
 \tkzDrawPoint[size=5pt,mark=x,color=red](A)
 \tkzDrawPoint[size=3pt,mark=*,color=blue](B)
\end{tikzpicture}
```

Example n° 30 Drawing defined points



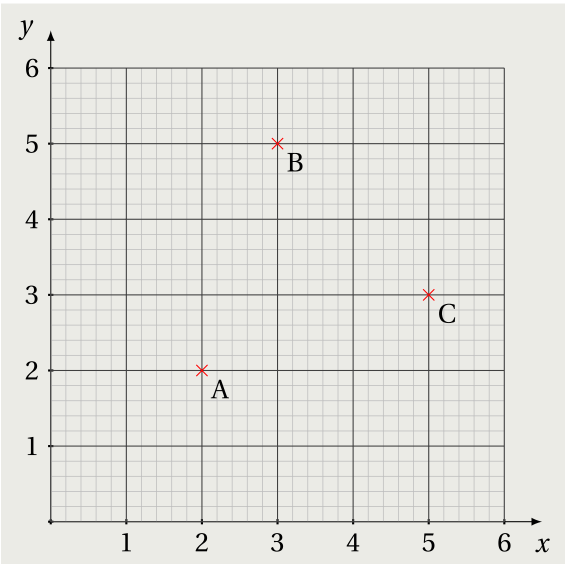

```
\begin{tikzpicture}
 \tkzInit[xmax=5,ymax=5]
 \tkzX
 \tkzY
 \tkzGrid[sub]
 \tkzPoint*(2,1){A}
 \tkzPoint*(4,3){B}
 \tkzPoint*(5,4){C}
 \tkzDrawPoint[size=5pt,mark=x,%
              color=red](A,B,C)
\end{tikzpicture}
```

macro n° 4 Set up local options for points `\SetUpPoint`

`\SetUpPoint[⟨local options⟩]`

It is possible to modify the default options for the points

Example n° 31 color,size and mark



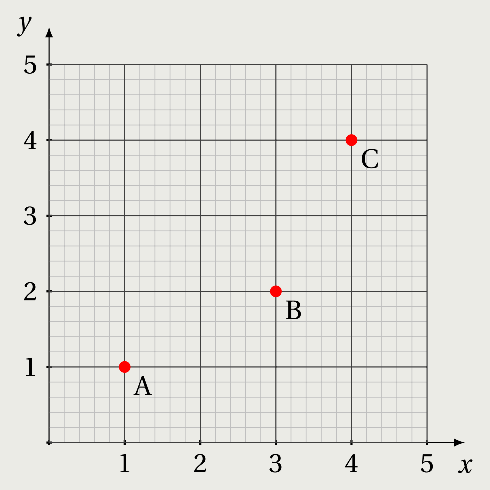

```
\begin{tikzpicture}
   \SetUpPoint[mark  = x,%
               size  = 3pt,%
               color = red]
   \tkzInit[xmax=6,ymax=6]
   \tkzX
   \tkzY
   \tkzGrid[sub]
   \tkzPoint(2,2){A}
   \tkzPoint(3,5){B}
   \tkzPoint(5,3){C}
 \end{tikzpicture}
```

macro n° 5 Set of points with names `\tkzPoints`

`\tkzPoints[⟨local options⟩]{⟨`$x_1/y_1/\mathrm{N}_1, x_2/y_2/\mathrm{N}_2$`,...⟩}`

You can obtain some points with the same local options. The elements of this list is composed with two cartesian coordinates for each point and the name of the point.

Example n° 32 Some points




```
\begin{tikzpicture}
   \tkzInit[xmax=5,ymax=5]
   \SetUpPoint[mark=*,size=2pt,color=red]
   \tkzX  \tkzY \tkzGrid[sub]
   \tkzPoints(%
              1/1/A,%
              3/2/B,%
              4/4/C)
\end{tikzpicture}
```

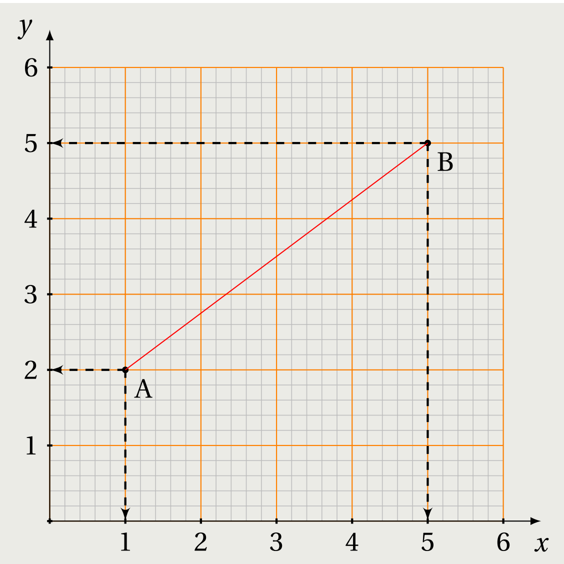
macro n° 6 Draw a segment or some segments\tkzSegment

\tkzSegment[⟨local options⟩](⟨A/B,...⟩)

options	défaut	définition
color	black	couleur du trait
lw	0.4pt	épaisseur du trait
style	solid	style du trait dashed,dotted etc...
symbol	{}	marque sur le segment
colorsymbol	black	couleur du symbol
namecolor	black	couleur du label
label	{}	étiquette du segment
colorlabel	black	nom de l'ordonnée si coord = true
poslabel	1pt	écart par rapport aux axes des coordonnées
time	.5	positionne le symbole et le label entre 0 et 1

You can draw one segment or some segments with the same local options. The elements of this list is composed with the name of two points. The names are separated by the symbol /.

Example n° 33 Simple segment



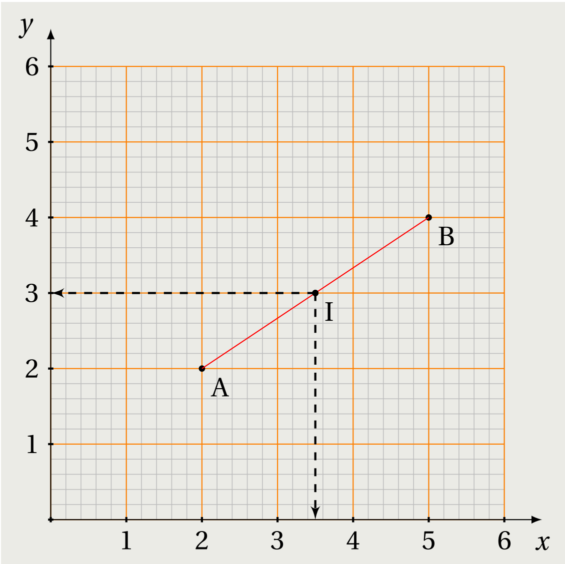

```
\begin{tikzpicture}
    \tkzInit[xmax=6,ymax=6]
    \tkzGrid[sub,color=orange]
    \tkzX
    \tkzY
    \tkzPoint[coord](1,2){A}
    \tkzPoint[coord](5,5){B}
    \tkzSegment[color=red](A/B)
\end{tikzpicture}
```

macro n° 7 Define the MidPoint of a segment `\tkzMidPoint`

`\tkzMidPoint[⟨local options⟩](⟨Point A,Point B⟩){⟨I⟩}`

I is the midpoint of the segment [AB]

Example n° 34 MidPoint of a segment



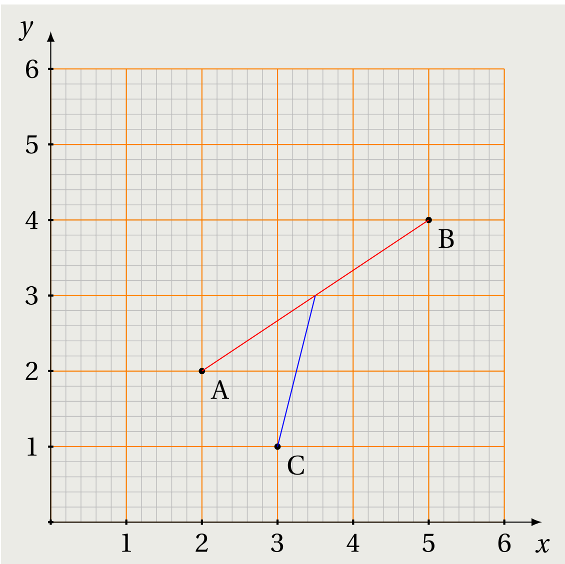

```
\begin{tikzpicture}
   \tkzInit[xmax=6,ymax=6]
   \tkzGrid[sub,color=orange]
   \tkzX
   \tkzY
   \tkzPoint(2,2){A}
   \tkzPoint(5,4){B}
   \tkzSegment[color=red](A/B)
   \tkzMidPoint[coord](A,B){I}
 \end{tikzpicture}
```

macro n° 8 Define the MidPoint of a segment `\tkzMidPoint*`

`\tkzMidPoint*[⟨local options⟩](⟨Point A,Point B⟩){⟨I⟩}`

I is the midpoint of the segment [AB] *but it is only defined and not drawn*

Example n° 35 How to use the midpoint of a segment



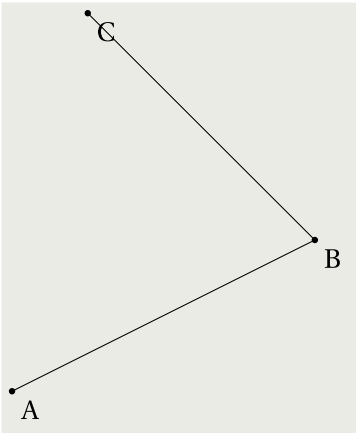

```
\begin{tikzpicture}
   \tkzInit[xmax=6,ymax=6]
   \tkzGrid[sub,color=orange]
   \tkzX
   \tkzY
   \tkzPoint(2,2){A}
   \tkzPoint(5,4){B}
   \tkzPoint(3,1){C}
   \tkzSegment[color=red](A/B)
   \tkzMidPoint*(A,B){I}
   \tkzSegment[color=blue](C/I)
\end{tikzpicture}
```

macro n° 9 Lines `\tkzPolySeg`

`\tkzPolySeg[⟨local options⟩](⟨Point A,Point B,...⟩)`

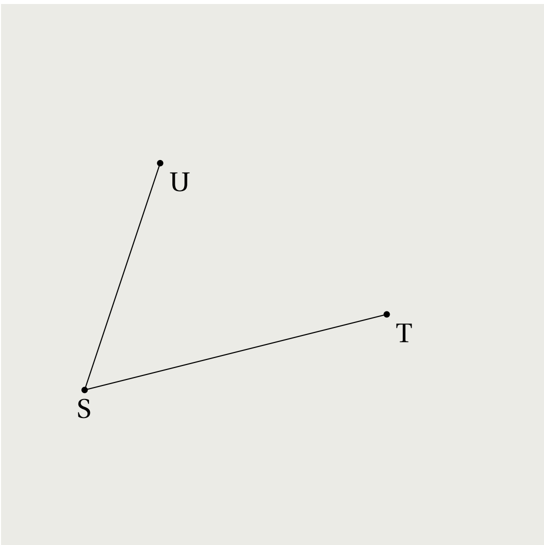
Draw or define (star-form) the intersection point I *of two lines* (AB) *and* (CD) *with point options.*

Example n° 36 Lines with segments




```
\begin{tikzpicture}
    \tkzInit[xmax=7,ymax=7]
    \tkzPoints%
           (2/1/A,%
            6/3/B,%
            3/6/C)
   \tkzPolySeg(A,B,C)
\end{tikzpicture}
```

Example n° 37 angle




```
\begin{tikzpicture}
   \tkzInit[xmax=7,ymax=7]
   \tkzPoint[pos=below](1,2){S}
   \tkzPoint(5,3){T}
   \tkzPoint(2,5){U}
   \tkzPolySeg(T,S,U)
   \tkzClip
   \path (S) circle (5cm);
\end{tikzpicture}
```

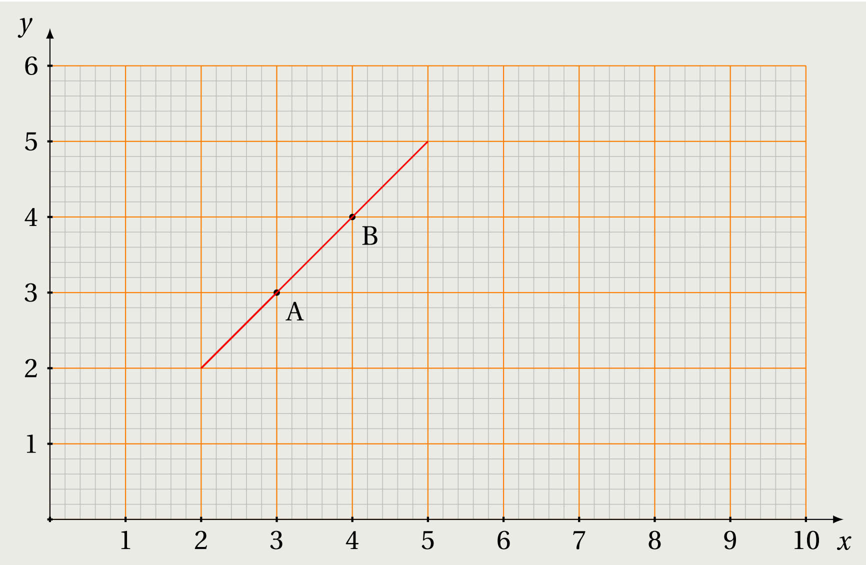
macro n° 10 Draw a Line `\tkzLine`

`\tkzLine[⟨local options⟩](⟨P_A/P_B,...⟩)`

options	défaut	définition
color	black	color of the line
lw	0.4pt	width of the line
style	solid	style of the line dashed,dotted etc...
prefix	d	prefix of the endpoints
kr	1	part of the line after AB, length=kr*AB
kl	1	part of the line before AB, length=kl*AB

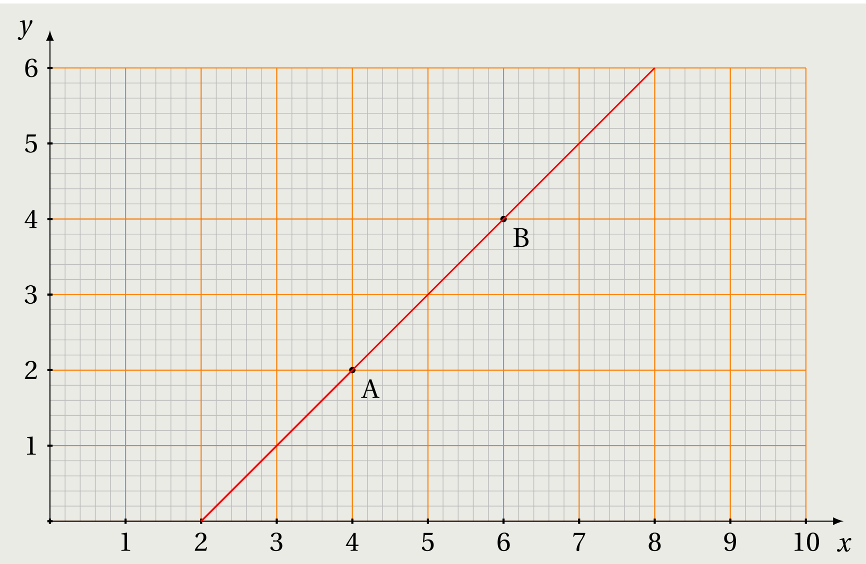
You can draw one line or some line with the same local options. The elements of this list is composed with the name of two points. The names are separated by the symbol /.

Example n° 38 A line



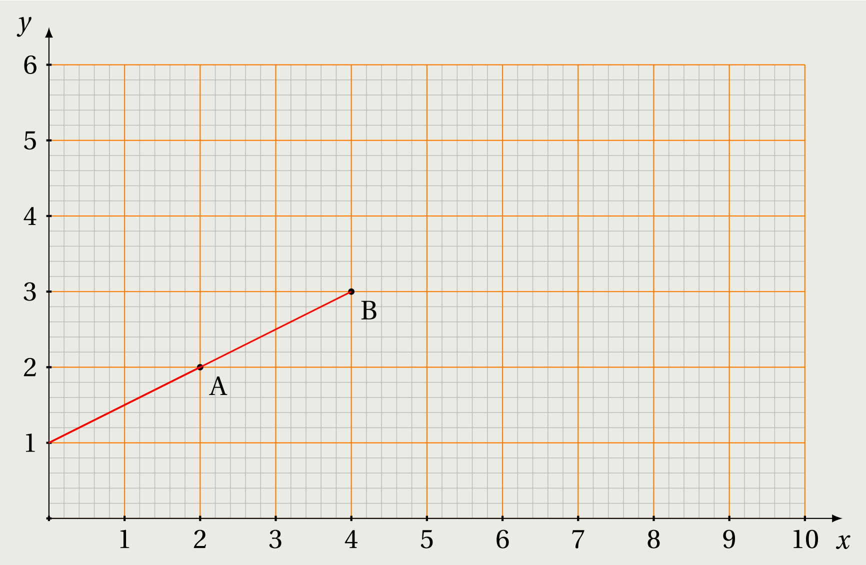

```
\begin{tikzpicture}
   \tkzInit[ymax=6]
   \tkzGrid[sub,color=orange]
   \tkzX
   \tkzY
   \tkzPoint(3,3){A}
   \tkzPoint(4,4){B}
   \tkzLine[color=red](A/B)
\end{tikzpicture}
```

Example n° 39 A line, options `kr`, `kl`



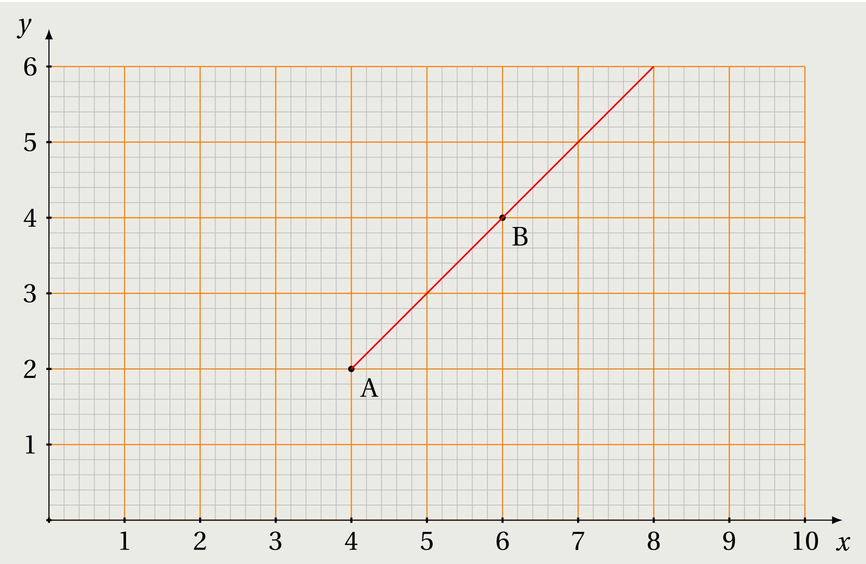

```
\begin{tikzpicture}
  \tkzInit[ymax=6]
  \tkzGrid[sub,color=orange]
  \tkzX
  \tkzY
  \tkzPoint(4,2){A}
  \tkzPoint(6,4){B}
  \tkzLine[color=red,kr=1,kl=1](A/B)
\end{tikzpicture}
```

Example n° 40 Half-line $kr = 0$



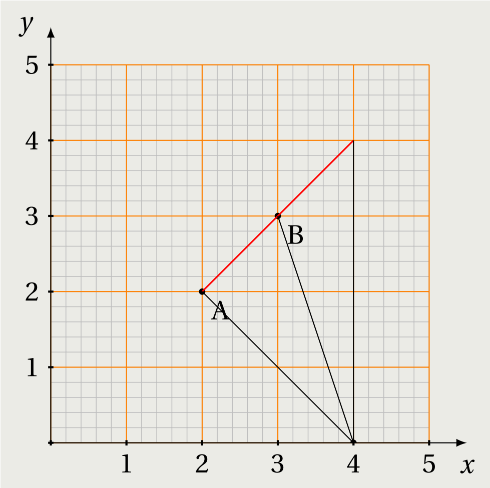

```
\begin{tikzpicture}
    \tkzInit[ymax=6]
    \tkzGrid[sub,color=orange]
    \tkzX
    \tkzY
    \tkzPoint(2,2){A}
    \tkzPoint(4,3){B}
    \tkzLine[color=red,kr=0,kl=1](A/B)
\end{tikzpicture}
```

Example n° 41 Half-line $kl = 0$




```
\begin{tikzpicture}
   \tkzInit[ymax=6]
   \tkzGrid[sub,color=orange]
   \tkzX
   \tkzY
   \tkzPoint(4,2){A}
   \tkzPoint(6,4){B}
   \tkzLine[color=red,kl=0,kr=1](A/B)
\end{tikzpicture}
```

Example n° 42 A line, option `prefix`



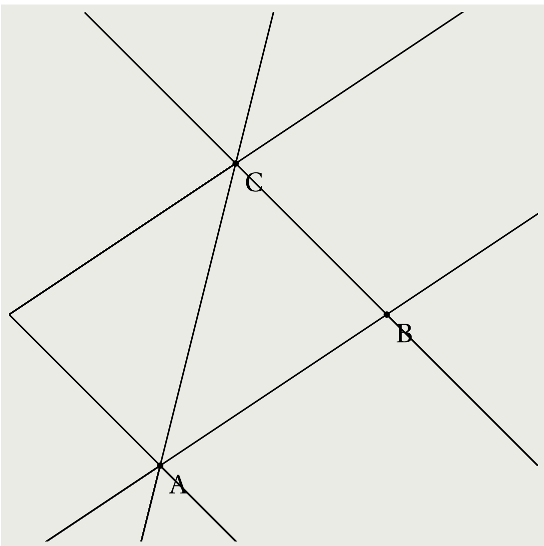

```
\begin{tikzpicture}
    \tkzInit[xmax=5,ymax=5]
    \tkzGrid[sub,color=orange]
    \tkzX
    \tkzY
    \tkzClip
    \tkzPoint(2,2){A}
    \tkzPoint(3,3){B}
    \tkzPoint(4,0){O}
    \tkzLine[color=red,kr=1,kl=0,%
             prefix=l](A/B)
    \tkzSegment(O/A,O/B,O/lr)
\end{tikzpicture}
```

macro n° 11 Parallel Lines `\tkzParaLL`

`\tkzParaLL[⟨local options⟩](⟨A,B⟩)(⟨C⟩)`

Draw a parallal line to (AB) *crossing* C.

Example n° 43 Parallel lines



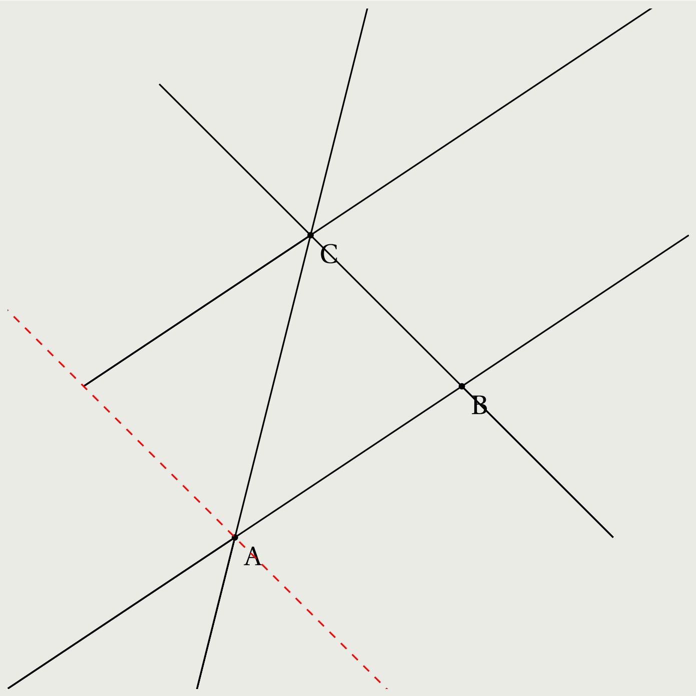

```
\begin{tikzpicture}
    \tkzInit[xmax=7,ymax=7]
    \tkzClip
    \tkzPoint(2,1){A}
    \tkzPoint(5,3){B}
    \tkzPoint(3,5){C}
    \tkzLine(A/B,B/C,A/C)
    \tkzParaLL(A,B)(C)
    \tkzParaLL[kr=1](B,C)(A)
\end{tikzpicture}
```

macro n° 12 Parallel Lines star-form `\tkzParaLL*`

`\tkzParaLL[⟨local options⟩](⟨A,B⟩)(⟨C⟩)`

Define a parallal line to (AB) *crossing* C.

Example n° 44 Parallel lines star-form



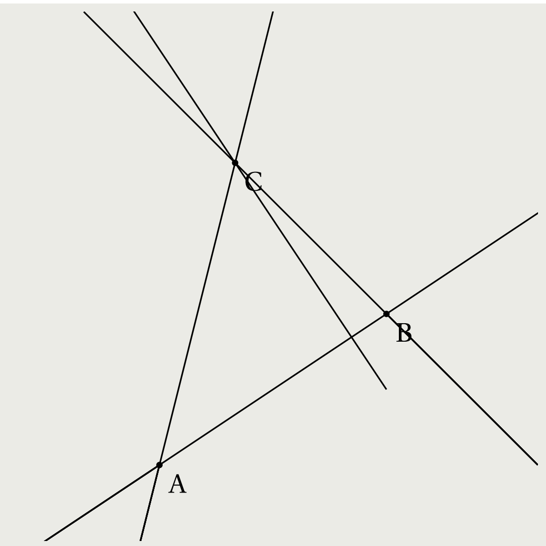

```
\begin{tikzpicture}
  \tkzInit[xmax=7,ymax=7]
  \tkzClip[space=1]
  \tkzPoint(2,1){A}
  \tkzPoint(5,3){B}
  \tkzPoint(3,5){C}
  \tkzLine(A/B,B/C,A/C)
  \tkzParaLL(A,B)(C)
  \tkzParaLL*[prefix=dd](B,C)(A)
  \tkzLine[color=red,%
           style=dashed](ddl/ddr)
\end{tikzpicture}
```

macro n° 13 Orthogonal Lines `\tkzLineOrth`

`\tkzLineOrth[⟨local options⟩](⟨A,B⟩)(⟨C⟩)`

Draw an orthogonal line to (AB) *crossing* C.

Example n° 45 Orthogonal lines




```
\begin{tikzpicture}
   \tkzInit[xmax=7,ymax=7]
   \tkzClip
   \tkzPoint(2,1){A}
   \tkzPoint(5,3){B}
   \tkzPoint(3,5){C}
   \tkzLine(A/B,B/C,A/C)
   \tkzLineOrth(A,B)(C)
\end{tikzpicture}
```

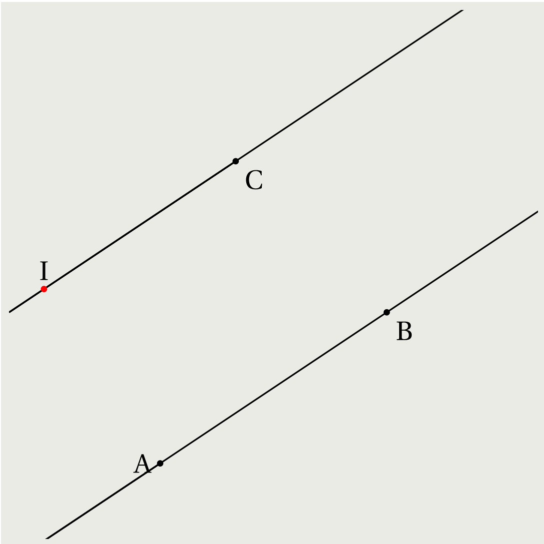
macro n° 14 Orthogonal Lines star-form `\tkzLineOrth*`

`\tkzLineOrth[⟨local options⟩](⟨A,B⟩)(⟨C⟩)`

Define (star-form) an orthogonal line to (AB) *crossing* C.

An other possibility is the projection of the point A.

Example n° 46 An orthogonal line (star-form)




```
\begin{tikzpicture}
   \tkzInit[xmax=7,ymax=7]
   \tkzClip
   \tkzPoint[pos=left](2,1){A}
   \tkzPoint(5,3){B}
   \tkzPoint(3,5){C}
   \tkzLine(A/B)
   \tkzParaLL[prefix=pl](A,B)(C)
   \tkzLineOrth*[prefix=or](A,B)(A)
   \tkzInterLL[%
       color = red,%
       pos = above](pll,plr)(orl,orr){I}
\end{tikzpicture}
```

macro n° 15 Define a circle of center O crossing A \tkzCircle

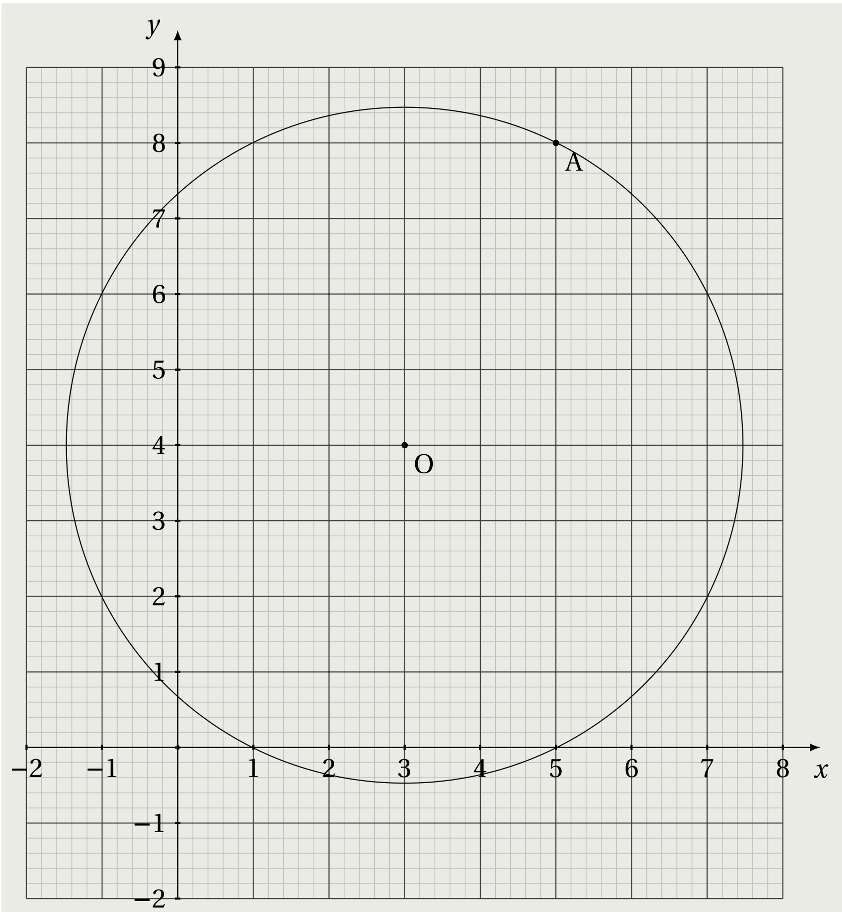
\tkzCircle[⟨*local options*⟩]{⟨O,A⟩}

options	défaut	définition
color	black	color of the line
lw	0.4pt	line width
style	solid	dashed,dotted etc...

A circle is defined with its center O *and a point of its circumference.*

Example n° 47 Circle with a center and a point of the circumference.

radius $r = \text{OA}$




```
\begin{tikzpicture}
 \tkzInit[xmin=-2,xmax=8,ymin=-2,ymax=9]
 \tkzGrid[sub]
 \tkzX
 \tkzY
 \tkzPoint(3,4){O}
 \tkzPoint(5,8){A}
 \tkzCircle(O,A)
\end{tikzpicture}
```

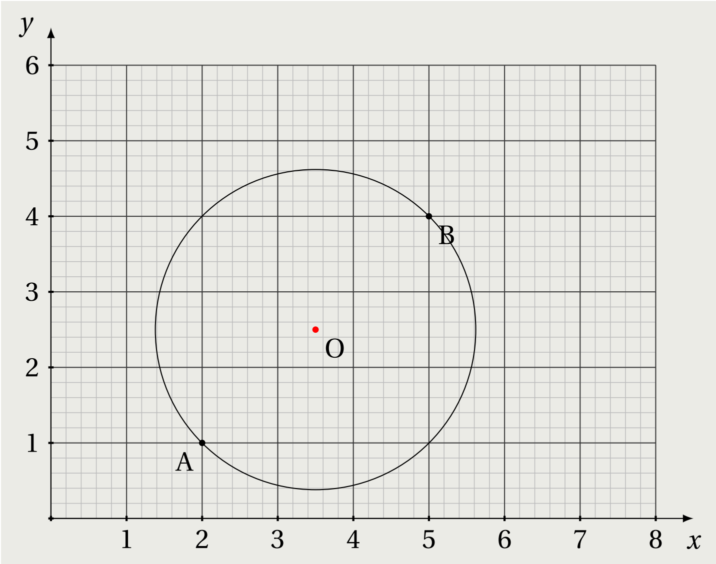
macro n° 16 Define a circle of diameter AB \tkzCircle*

\tkzCircle*[⟨*local options*⟩]{⟨A,B⟩}

A circle is defined with two diameterly opposed points.

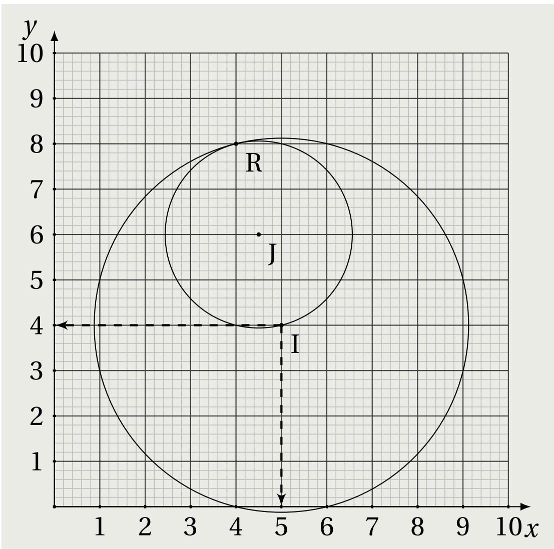
Example n° 48 Circle with two diameterly opposed points.

diameter $d = \mathrm{AB}$




```
\begin{tikzpicture}[scale=1]
   \tkzInit[xmax=8,ymax=6]
   \tkzGrid[sub]
   \tkzX
   \tkzY
   \tkzPoint[pos=below left](2,1){A}
   \tkzPoint(5,4){B}
   \tkzMidPoint[color=red](A,B){O}
   \tkzCircle*(A,B)
\end{tikzpicture}
```

Example n° 49 Circles : the two forms




```
\begin{tikzpicture}[scale=.6]
   \tkzInit
   \tkzGrid[sub]
   \tkzX\tkzY
   \tkzPoint*(2,6){S}
   \tkzPoint*(8,2){T}
   \tkzMidPoint[coord](S,T){I}
   \tkzPoint(4,8){R}
   \tkzMidPoint(I,R){J}
   \tkzCircle(I,R)
   \tkzCircle*(I,R)
\end{tikzpicture}
```

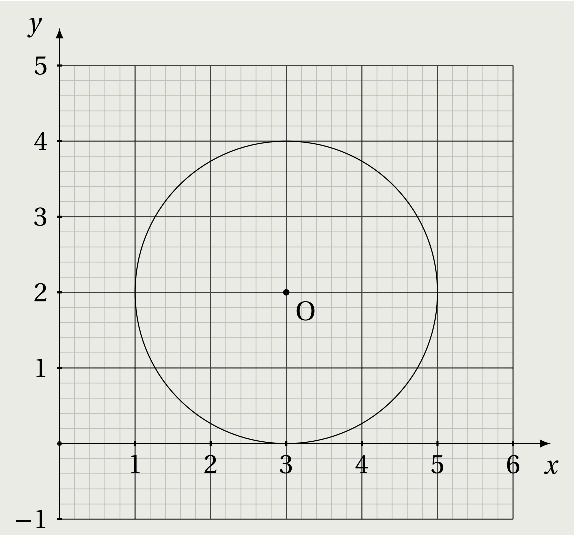
macro n° 17 Define a circle of center A and a Radius `\tkzCircleR`

`\tkzCircleR[⟨local options⟩]{⟨O,r⟩}`

A circle is defined with its center O *and radius r (unit=cm).*

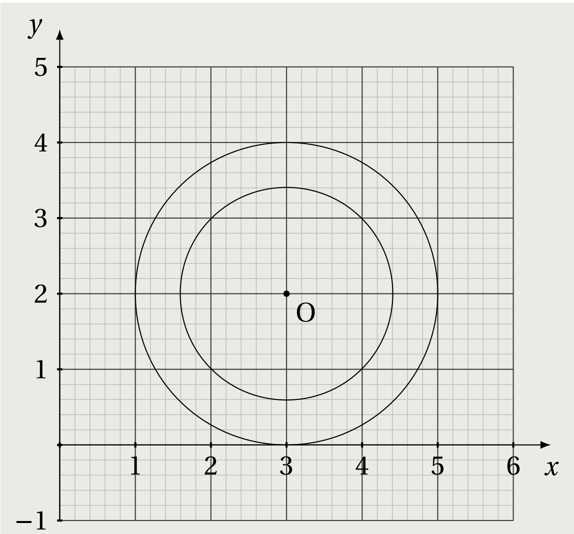
Example n° 50 Circle with a radius and `\tkzrayon`

`\tkzrayon` is a pre-defined dimension register.




```
\begin{tikzpicture}[scale=1]
   \tkzInit[xmax=6,ymin=-1,ymax=5]
   \tkzGrid[sub]
   \tkzX
   \tkzY
   \tkzrayon=2cm
   \tkzPoint(3,2){O}
   \tkzCircleR(O,\tkzrayon)
\end{tikzpicture}
```

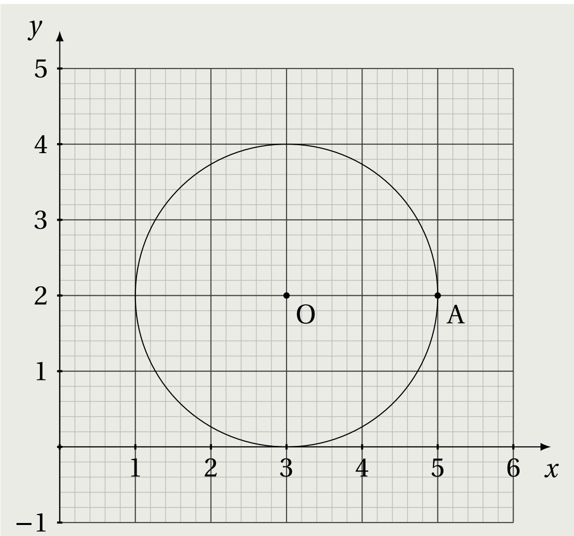
Example n° 51 Circle with a radius with cm or pt




```
\begin{tikzpicture}[scale=1]
  \tkzInit[xmax=6,ymin=-1,ymax=5]
  \tkzGrid[sub]
  \tkzX
  \tkzY
  \tkzPoint(3,2){O}
  \tkzCircleR(O,2 cm)\tkzCircleR(O,40 pt)
\end{tikzpicture}
```

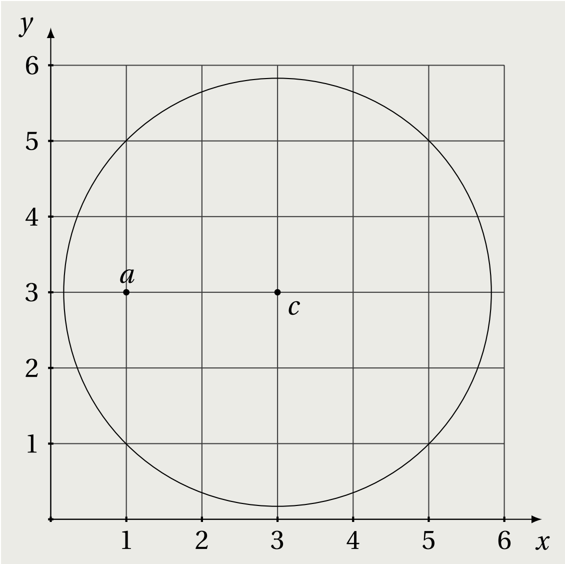
Example n° 52 Circle with `\tkzMathLength`

`\tkzMathLength` is a macro to obtain the length of a segment. The result is in `\tkzmathLen`. See below, how to use this macro.




```
\begin{tikzpicture}[scale=1]
  \tkzInit[xmax=6,ymin=-1,ymax=5]
  \tkzGrid[sub]
  \tkzX
  \tkzY
  \tkzPoint(3,2){O}\tkzPoint(5,2){A}
  \tkzMathLength(O,A)
  \tkzCircleR(O,\tkzmathLen pt)
\end{tikzpicture}
```

Example n° 53 Circle with pgfmath



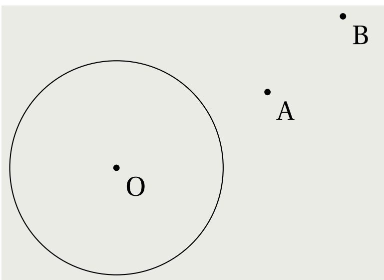

```
\begin{tikzpicture}
   \tkzInit[xmax=6,ymax=6]
   \tkzGrid\tkzX\tkzY
   \tkzPoint*(0,0){o}
   \tkzPoint(3,3){c}
   \tkzPoint[pos=above](1,3){a}
   \pgfmathparse{2*sqrt(2)}
   \tkzrayon=\pgfmathresult cm
   \tkzCircleR(c,\tkzrayon)
\end{tikzpicture}
```

macro n° 18 Define a circle of center A and a Radius given by two points. `\tkzCircleR*`

`\tkzCircleR*[⟨local options⟩]{⟨O,A,B⟩}`

A circle is defined with its center O *and radius given by two points.*

Example n° 54 Circle with a radius given by two points




```
\begin{tikzpicture}
  \tkzInit[xmax=6,ymax=6]
  \tkzPoint(3,2){O}
  \tkzPoint(5,3){A}
  \tkzPoint(6,4){B}
  \tkzCircleR*(O,A,B)
\end{tikzpicture}
```

VII . Special Objects

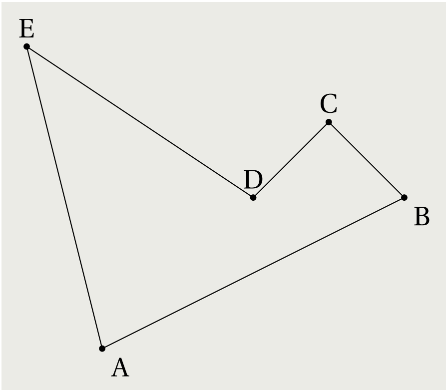
macro n° 19 polygon `\tkzPolygon`

`\tkzPolygon[⟨local options⟩](⟨A,B,...⟩)`

options	default	definition
color	black	color of the line
lw	0.4pt	width of the line
style	solid	style of the line

Draw a polygon with a list of points.

Example n° 55 polygon



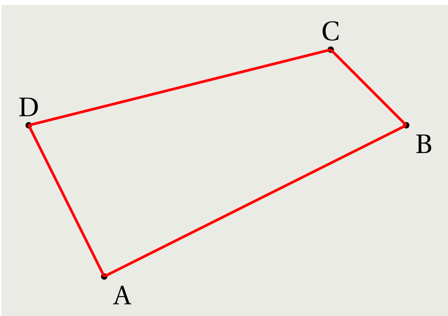

```
\begin{tikzpicture}
   \tkzInit[xmin=-1,ymin=-1,xmax=7,ymax=7]
   \tkzPoint(2,1){A}
   \tkzPoint(6,3){B}
   \tkzPoint[pos=above](5,4){C}
   \tkzPoint[pos=above](4,3){D}
   \tkzPoint[pos=above](1,5){E}
   \tkzPolygon(A,B,C,D,E)
\end{tikzpicture}
```

macro n° 20 Set up pline `\SetUpMLine`

`\SetUpMLine[⟨local options⟩]`

This macro can be used to change default values (see above)

Example n° 56 Set Up Line for Polygons



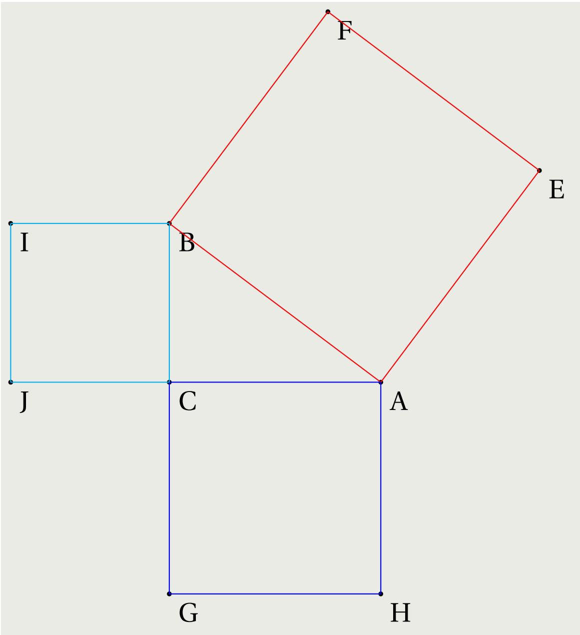

```
\begin{tikzpicture}
   \tkzInit[xmin=-1,ymin=-1,xmax=7,ymax=7]
   \tkzPoint(2,1){A}
   \tkzPoint(6,3){B}
   \tkzPoint[pos=above](5,4){C}
   \tkzPoint[pos=above](1,3){D}
   \tkzSetUpPLine[color=red,lw=1pt]
   \tkzPolygon(A,B,C,D)
  \end{tikzpicture}
```

macro n° 21 Square \tkzSquare

\tkzSquare[⟨local options⟩](⟨ A,B⟩){⟨C⟩}{⟨D⟩}

ABCD *is a square with* $\overrightarrow{\mathrm{BA}}$ *orthogonal to* $\overrightarrow{\mathrm{BC}}$ *(+90°).*
`Local options` *are the same ones as for* \tkzPolygon.

Example n° 57 Squares



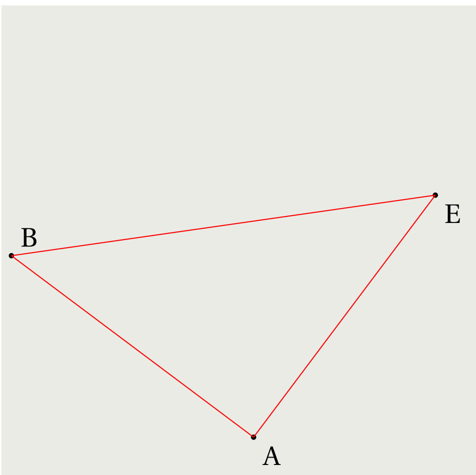

```
\begin{tikzpicture}[scale=.7]
   \tkzInit
   \tkzPoint(0,0){C}
   \tkzPoint(4,0){A}
   \tkzPoint(0,3){B}
   \tkzSquare[color=red](B,A){E}{F}
   \tkzSquare[color=blue](A,C){G}{H}
   \tkzSquare[color=cyan](C,B){I}{J}
\end{tikzpicture}
```

macro n° 22 Square star-form \tkzSquare*

\tkzSquare*(⟨ A,B⟩){⟨C⟩}{⟨D⟩}

The points C *and* D *are defined and that's all!*

Example n° 58 Square star-form isosceles right triangle



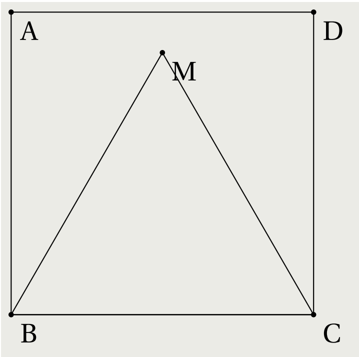

```
\begin{tikzpicture}[scale=.8]
   \tkzInit[xmax=6,ymax=6]
   \tkzPoint*(0,0){C}
   \tkzPoint(4,0){A}
   \tkzPoint[pos=above right](0,3){B}
   \tkzSquare*(B,A){E}{F}
   \tkzDrawPoint(E)
   \tkzPolygon[color=red](A,B,E)
\end{tikzpicture}
```

macro n° 23 Equilateral triangle \tkzTrEqui

\tkzTrEqui[⟨*local options*⟩](⟨A, B⟩){⟨C⟩}

ABC *is an equilateral triangle with an angle of* 60° *between* $\overrightarrow{BA}$ *and* $\overrightarrow{BC}$.
Local options *are the same ones as for* *\tkzPolygon*.

Example n° 59 Equilateral triangle



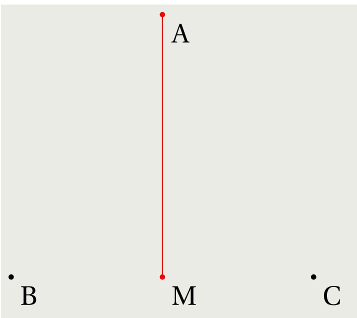

```
\begin{tikzpicture}[scale=0.8]
   \tkzInit[xmax=6,ymax=6]
   \tkzPoint(0,0){B}
   \tkzPoint(5,0){C}
   \tkzSquare(B,C){D}{A}
   \tkzTrEqui(B,C){M}
 \end{tikzpicture}
```

macro n° 24 Equilateral triangle star-form \tkzTrEqui*

\tkzTrEqui*(⟨A, B⟩){⟨C⟩}

The point C *is defined and that's all.*

Example n° 60 Equilateral triangle star-form




```
\begin{tikzpicture}[scale=0.8]
   \tkzInit[xmax=6,ymax=6]
   \tkzPoint(0,0){B}
   \tkzPoint(5,0){C}
   \tkzTrEqui*(B,C){A}
   \tkzMidPoint[color=red](B,C){M}
   \tkzDrawPoint[color=red](A)
   \tkzSegment[color=red](A/M)
 \end{tikzpicture}
```

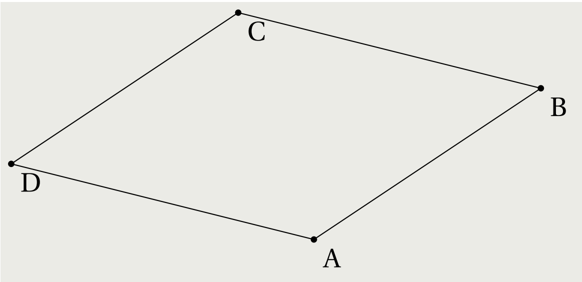
macro n° 25 Parallelogram `\tkzLLgram`

`\tkzLLgram[⟨local options⟩](⟨ A,B,C⟩){⟨D⟩}`

$\overrightarrow{AB}$ *is collinear to* $\overrightarrow{CD}$.

`Local options` *are the same ones as for* `\tkzPolygon`.

Example n° 61 Parallelogram




```
\begin{tikzpicture}
   \tkzInit[xmax=6,ymax=6]
   \tkzPoint(2,1){A}
   \tkzPoint(5,3){B}
   \tkzPoint(1,4){C}
   \tkzLLgram(A,B,C){D}
\end{tikzpicture}
```

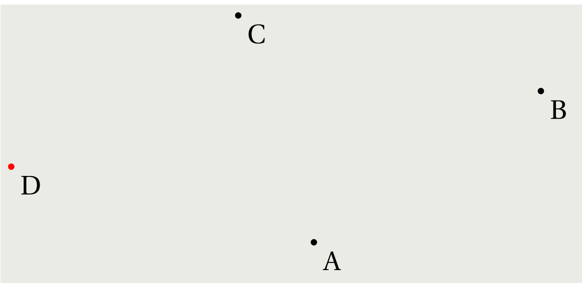
macro n° 26 Parallelogram star-form `\tkzLLgram*`

`\tkzLLgram*(⟨ A,B,C⟩){⟨D⟩}`

$\overrightarrow{AB}$ *is collinear to* $\overrightarrow{CD}$.

`Local options` *are the same ones as for* `\tkzPolygon`.

Example n° 62 Parallelogram star-form




```
\begin{tikzpicture}
   \tkzInit[xmax=6,ymax=6]
   \tkzPoint(2,1){A}
   \tkzPoint(5,3){B}
   \tkzPoint(1,4){C}
   \tkzLLgram*(A,B,C){D}
   \tkzDrawPoint[color=red](D)
\end{tikzpicture}
```

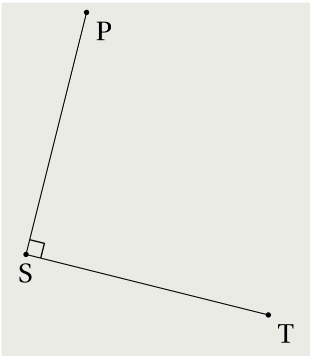
macro n° 27 Mark Right Angle \tkzRightAngle

\tkzRightAngle[⟨*local options*⟩](⟨A,B,C⟩)

options	default	definition
color	black	color of the line
size	0.25	radius of the arc

Draw a little square to mark the right angle

Example n° 63 Mark Right Angle




```
\begin{tikzpicture}[scale=.8]
 \tkzInit[xmax=7,ymax=5]
 \tkzPoint[pos=below](1,2){S}
 \tkzPoint(5,1){T}
 \tkzRotate(S,90)(T/P)
 \tkzRightAngle(T/S/P)
 \tkzSegment(S/T,S/P)
\end{tikzpicture}
```

macro n° 28 Mark Angle \tkzMarkAngle

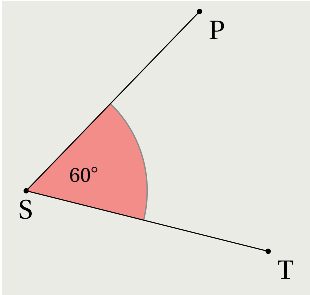
\tkzMarkAngle[⟨*local options*⟩](⟨A/B/C⟩)

options	default	definition
color	black	color of the line
fillcolor	white	color of the sector
size	1cm	radius of the arc
style	ai	ai, aii or aiii (see below)
label	{}	symbol for the angle
labelcolor	1	part after D
labelsize	\scriptstyle	size of the label
pos	{}	above, left etc ..
dist	0.5	coefficient to place the label

```
\tikzstyle{ai}=[draw,line width=.5pt]
\tikzstyle{aii}=[draw,line width=.5pt,double,double distance=1pt]
\tikzstyle{aiii}=[draw,line width=.5pt,double=red,double distance=2pt]
```

You can modify the styles "ai", "aii" and "aiii". You must use `\label` *without $ $. The mathematic mode is always set. The option "dist" is special, it's a number. You adjust it but if* $dist = 1$ *then the label is placed at a distance equal at "len" (see option "len") of bisector, bu default* $len = 5cm$.

Example n° 64 Mark Angle




```
\begin{tikzpicture}[scale=.8]
 \tkzInit[xmax=7,ymax=5]
 \tkzPoint[pos=below](1,2){S}
 \tkzPoint(5,1){T}
 \tkzRotate(S,60)(T/P)
 \tkzMarkAngle[size      = 2,%
              style     = ai,%
              dist      = .2,%
              label     = 60^{\circ},%
              color     = black,%
              fillcolor = red](T/S/P)
 \tkzSegment(S/T,S/P)
\end{tikzpicture}
```

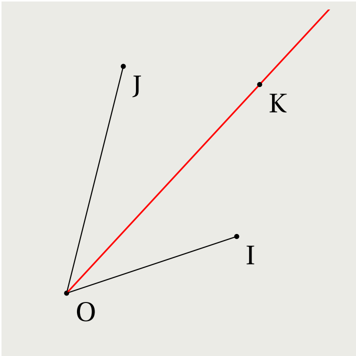
macro n° 29 Bisector `\tkzBisector`

`\tkzBisector[⟨local options⟩](⟨A,B,C⟩){⟨D⟩}`

options	default	definition
color	black	color of the line
lw	0.6pt	width of the line
style	solid	style of the line
kl	1	part before O
kr	1	part after D
len	5	length of BD
prefix	d	dl and dr are points of the line

Draw or define (star-form) the (interior) bisector (also called the internal angle bisector) of an angle $\widehat{\mathrm{ABC}}$.
The point D *is a point of the bisector and* BD=`len`.

Example n° 65 Bisector



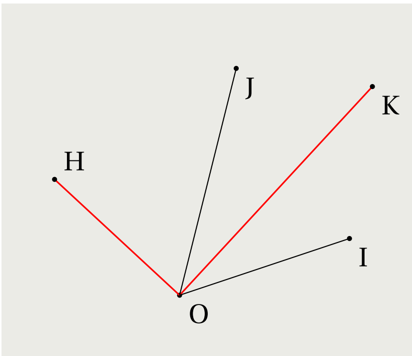

```
\begin{tikzpicture}[scale=.75]
  \tkzInit[xmin=-1,xmax=5,ymin=-1,ymax=5]
  \tkzClip
  \tkzPoint(0,0){O}
  \tkzPoint(3,1){I}
  \tkzPoint(1,4){J}
  \tkzBisector[color=red,kl=0](I,O,J){K}
  \tkzDrawPoint(K)
  \tkzSegment(O/I,O/J)
\end{tikzpicture}
```

macro n° 30 Bisector `\tkzBisectorOut`

`\tkzBisectorOut[⟨local options⟩](⟨A,B,C⟩){⟨D⟩}`

Draw or define (star-form) the (exterior) bisector (also called the exterior angle bisector) of an angle $\widehat{ABC}$. *The point* D *is a point of the bisector. The* `len option` *gives the length of* BD.

Example n° 66 Bisector Out



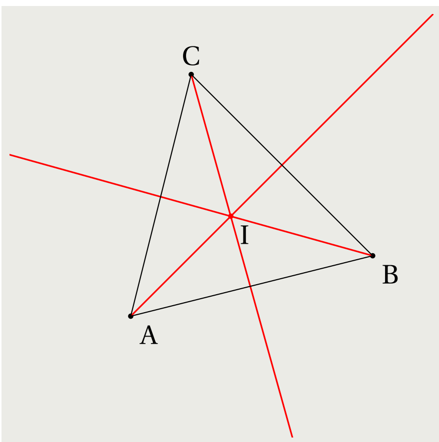

```
\begin{tikzpicture}[scale=.75]
  \tkzInit[xmin=-3,xmax=4,ymin=-1,ymax=5]
  \tkzClip
  \tkzPoint(0,0){O}
  \tkzPoint(3,1){I}
  \tkzPoint(1,4){J}
  \tkzBisector[color=red,%
               kl=0,kr=0](I,O,J){K}
  \tkzDrawPoint(K)
  \tkzBisectorOut[color=red,%
                  kl=0,kr=0,len=3](I,O,J){H}
  \tkzDrawPoint[pos=above right](H)
  \tkzSegment(O/I,O/J)
\end{tikzpicture}
```

macro n° 31 InCenter `\tkzInCenter`

`\tkzInCenter[⟨local options⟩](⟨A,B,C⟩){⟨I⟩}`

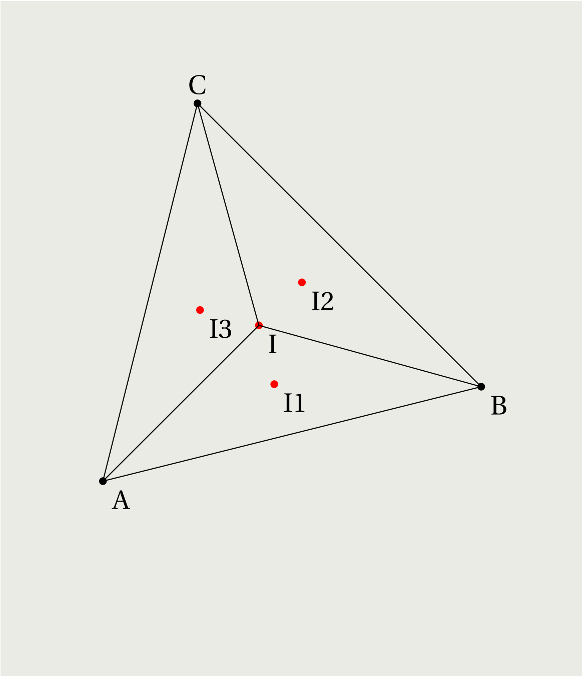
The angle bisectors of the triangle ABC *meet at the incenter* I

Example n° 67 InCenter with the bisectors




```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=7,ymax=7]
  \tkzClip
  \tkzPoint(2,2){A}
  \tkzPoint(6,3){B}
  \tkzPoint[pos=above](3,6){C}
  \tkzBisector[color=red,kl=0](A,B,C){I}
  \tkzBisector[color=red,kl=0](B,A,C){J}
  \tkzBisector[color=red,kl=0](B,C,A){K}
  \tkzInCenter[color=red,size=1pt](A,B,C){I}
  \tkzPolygon(A,B,C)
\end{tikzpicture}
```

Example n° 68 InCenters



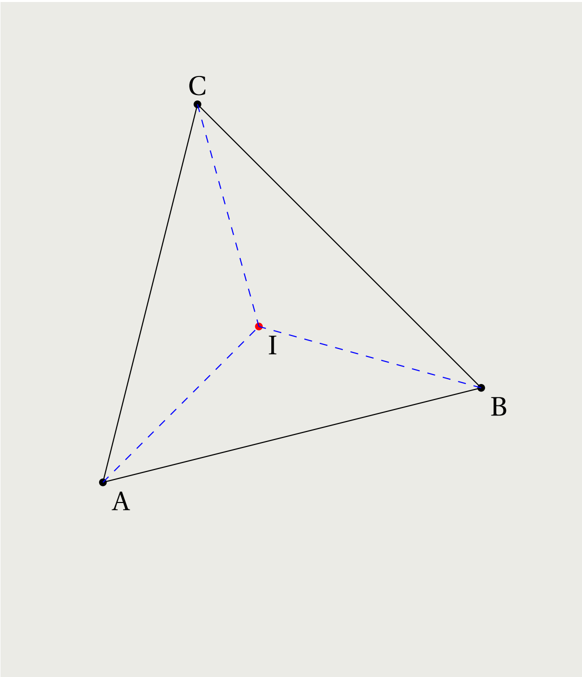

```
\begin{tikzpicture}[scale=1.25]
  \tkzInit[xmax=6,ymax=7]
  \tkzClip
  \tkzPoint(1,2){A}
  \tkzPoint(5,3){B}
  \tkzPoint[pos=above](2,6){C}
  \tkzInCenter[color=red,%
               size=1pt](A,B,C){I}
  \tkzPolygon(A,B,C)
  \tkzSegment(A/I,B/I,C/I)
  \tkzInCenter[color = red,%
               size  = 1pt](A,B,I){I1}
  \tkzInCenter[color = red,%
               size  = 1pt](C,B,I){I2}
  \tkzInCenter[color = red,%
               size  = 1pt](A,C,I){I3}
\end{tikzpicture}
```

macro n° 32 InCenter star-form `\tkzInCenter*`

`\tkzInCenter[⟨local options⟩](⟨A,B,C⟩)`

Local option equal point options.

Example n° 69 InCenter star-form

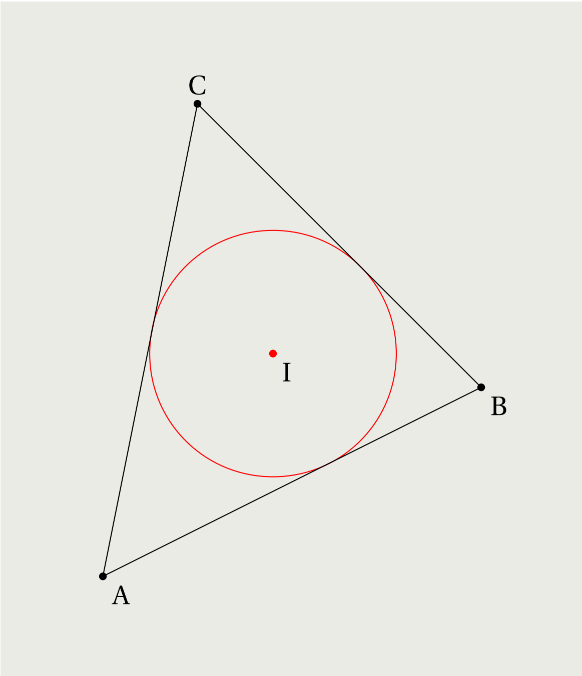



```
\begin{tikzpicture}[scale=1.25]
  \tkzInit[xmin=1,xmax=7,ymax=7]
  \tkzClip
  \tkzPoint(2,2){A}
  \tkzPoint(6,3){B}
  \tkzPoint[pos=above](3,6){C}
  \tkzInCenter[color=red](A,B,C){I}
  \tkzPolygon(A,B,C)
  \tkzSegment[style=dashed,%
              color=blue](A/I,B/I,C/I)
\end{tikzpicture}
```

macro n° 33 InCircle `\tkzInCircle`

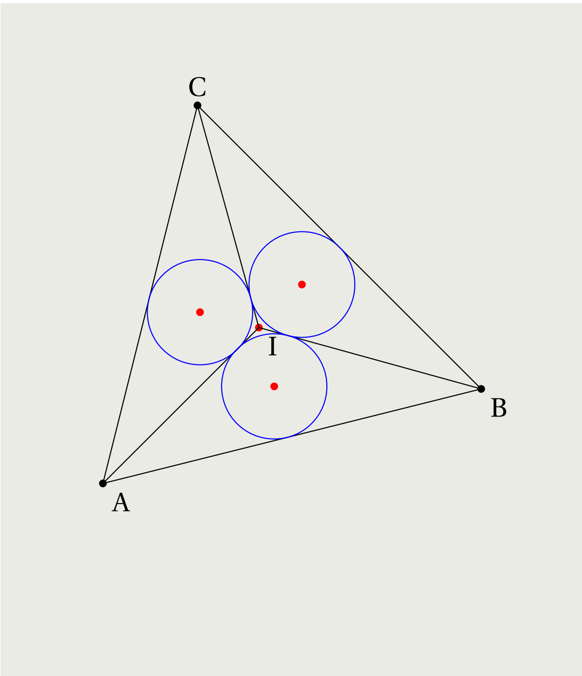
`\tkzInCircle[⟨local options⟩](⟨A,B,C⟩)`

Example n° 70 InCircle




```
\begin{tikzpicture}[scale=1.25]
  \tkzInit[xmax=6,ymax=7]
  \tkzClip
  \tkzPoint(1,1){A}
  \tkzPoint(5,3){B}
  \tkzPoint[pos=above](2,6){C}
  \tkzInCenter[color = red,%
               size  = 1pt](A,B,C){I}
  \tkzInCircle[color=red](A,B,C)
  \tkzPolygon(A,B,C)
\end{tikzpicture}
```

Example n° 71 InCircles



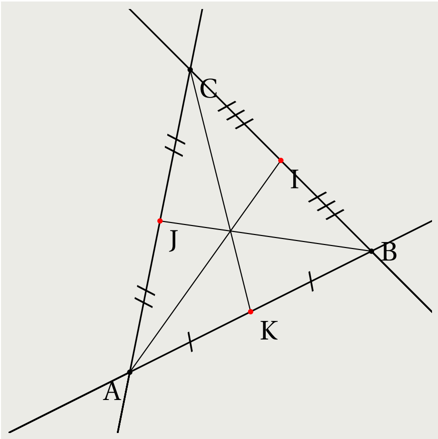

```
\begin{tikzpicture}[scale=1.25]
  \tkzInit[xmax=6,ymax=7]
  \tkzClip
  \tkzPoint(1,2){A}
  \tkzPoint(5,3){B}
  \tkzPoint[pos=above](2,6){C}
  \tkzInCenter[color=red,%
               size = 1pt](A,B,C){I}
  \tkzPolygon(A,B,C)
  \tkzSegment(A/I,B/I,C/I)
  \tkzInCenter[color= red,%
              size  = 1pt,%
                      noname](A,B,I){I1}
  \tkzInCenter[color = red,%
               size  = 1pt,%
                       noname](C,B,I){I2}
  \tkzInCenter[color = red,%
               size  = 1pt,%
               noname](A,C,I){I3}
  \tkzInCircle[color=blue](A,B,I)
  \tkzInCircle[color=blue](A,C,I)
  \tkzInCircle[color=blue](B,C,I)
\end{tikzpicture}
```

macro n° 34 Median `\tkzMedian`

`\tkzMedian[⟨local options⟩](⟨A,B⟩)(⟨C⟩)`

Median from C *to the middle of* [AB]. *Local options equal line options.*

Example n° 72 Medians



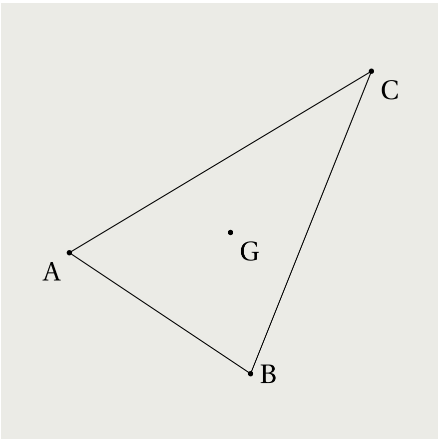

```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=7,ymax=7]
  \tkzClip
  \tkzPoint[pos=below left](2,1){A}
  \tkzPoint[pos=right](6,3){B}
  \tkzPoint(3,6){C}
  \tkzLine(A/B,B/C,A/C)
  \tkzMedian(A,B)(C)
  \tkzMedian(B,C)(A)
  \tkzMedian(A,C)(B)
  \tkzMidPoint[color=red](A,B){K}
  \tkzMidPoint[color=red](A,C){J}
  \tkzMidPoint[color=red](B,C){I}
  \tkzSegmentMark[symbol=/](A/K,B/K)
  \tkzSegmentMark[symbol=//](A/J,C/J)
  \tkzSegmentMark[symbol=///](B/I,C/I)
\end{tikzpicture}
```

macro n° 35 Gravity Center `\tkzGravityCenter`

`\tkzGravityCenter[⟨local options⟩](⟨ A,B,C⟩){⟨G⟩}`

Draw or define (star-form) the gravity center of a triangle. Local options equal point options.

Example n° 73 The Gravity Center



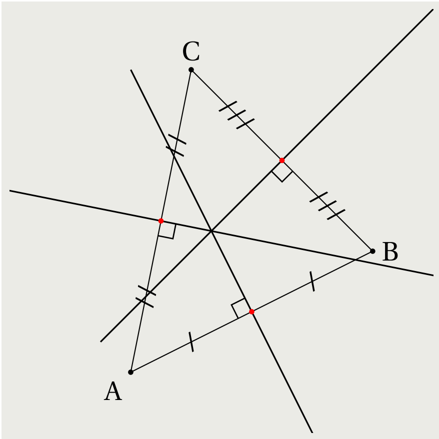

```
  \begin{tikzpicture}[scale=.8]
      \tkzInit[xmax=7,ymax=7]
      \tkzClip
      \tkzPoint[pos=below left](1,3){A}
      \tkzPoint[pos=right](4,1){B}
      \tkzPoint(6,6){C}
      \tkzGravityCenter(A,B,C){G}
      \tkzPolygon(A,B,C)
  \end{tikzpicture}
```

macro n° 36 MediatorLine `\tkzMediatorLine`

`\tkzMediatorLine[⟨local options⟩](⟨A,B⟩)`

Draw or define (star-form) the mediator line of [AB]. *Local options equal line options.*

Example n° 74 Mediator Line



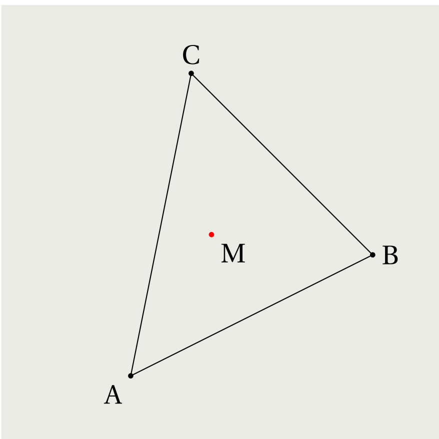

```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=7,ymax=7]\tkzClip
  \tkzPoint[pos=below left](2,1){A}
  \tkzPoint[pos=right](6,3){B}
  \tkzPoint[pos=above](3,6){C}
  \tkzPolygon(A,B,C)
  \tkzMediatorLine[prefix=m1](A,B)
  \tkzRightAngle(A/tkzmidpoint/m1r)
  \tkzDrawPoint[color=red,noname](tkzmidpoint)
  \tkzSegmentMark[symbol=/](A/tkzmidpoint,%
                            B/tkzmidpoint)
  \tkzMediatorLine[prefix=m2](A,C)
  \tkzRightAngle(A/tkzmidpoint/m2l)
  \tkzDrawPoint[color=red,noname](tkzmidpoint)
  \tkzSegmentMark[symbol=//](A/tkzmidpoint,%
                            C/tkzmidpoint)
  \tkzMediatorLine[prefix=m3](C,B)
  \tkzRightAngle(B/tkzmidpoint/m3l)
  \tkzDrawPoint[color=red,noname](tkzmidpoint)
  \tkzSegmentMark[symbol=///](B/tkzmidpoint,%
                              C/tkzmidpoint)
\end{tikzpicture}
```

macro n° 37 CircumCenter `\tkzCircumCenter`

`\tkzCircumCenter[⟨local options⟩](⟨A,B,C⟩)`

Draw or define (star-form) the circumcenter of a triangle. Local options equal point options.

Example n° 75 CircumCenter



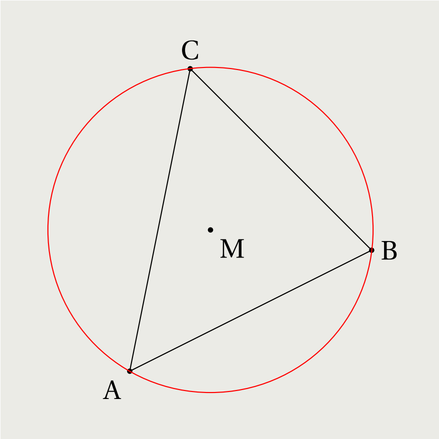

```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=7,ymax=7]\tkzClip
  \tkzPoint[pos=below left](2,1){A}
  \tkzPoint[pos=right](6,3){B}
  \tkzPoint[pos=above](3,6){C}
  \tkzPolygon(A,B,C)
  \tkzCircumCenter[color=red,size=1pt](A,B,C){M}
\end{tikzpicture}
```

macro n° 38 CircumCircle `\tkzCircumCircle`

`\tkzCircumCircle[⟨local options⟩](⟨A,B,C⟩){⟨M⟩}`

M *is the CircumCenter of the triangle* ABC. *Local options equal polygon options.*

Example n° 76 CircumCircle



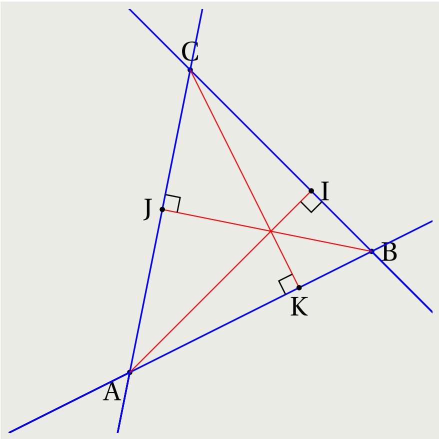

```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=7,ymax=7]\tkzClip
  \tkzPoint[pos=below left](2,1){A}
  \tkzPoint[pos=right](6,3){B}
  \tkzPoint[pos=above](3,6){C}
  \tkzPolygon(A,B,C)
  \tkzCircumCenter(A,B,C){M}
  \tkzCircumCircle[color=red](A,B,C)
\end{tikzpicture}
```

macro n° 39 Altitude `\tkzAltitude`

`\tkzAltitude[⟨local options⟩](⟨ A,B⟩)(⟨C/P⟩)`

P *is the projection of* C *on* [AB]. *Local options equal line options.*

Example n° 77 Altitudes



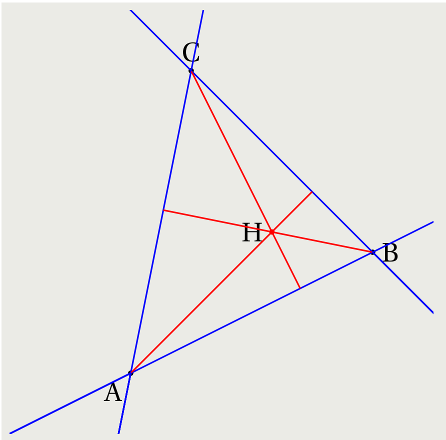

```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=7,ymax=7]
  \tkzClip
  \SetUpPoint[pos=right]
  \tkzPoint[pos=below left](2,1){A}
  \tkzPoint[pos=right](6,3){B}
  \tkzPoint[pos=above](3,6){C}
  \tkzLine[color=blue](A/B,B/C,A/C)
  \SetUpPoint[pos=left]
  \tkzAltitude[color=red](A,B)(C/K)
  \tkzAltitude[color=red](A,C)(B/J)
  \tkzAltitude[color=red](C,B)(A/I)
  \tkzRightAngle(A/K/C,B/I/A,C/J/B)
  \tkzDrawPoint[pos=right](I)
  \tkzDrawPoint[pos=left](J)
  \tkzDrawPoint[pos=below](K)
 \end{tikzpicture}
```

macro n° 40 OrthoCenter `\tkzOrthoCenter`

`\tkzOrthoCenter[⟨local options⟩](⟨A,B,C⟩){⟨H⟩}`

Draw or define (star-form) the orto center of a triangle. Local options equal point options.

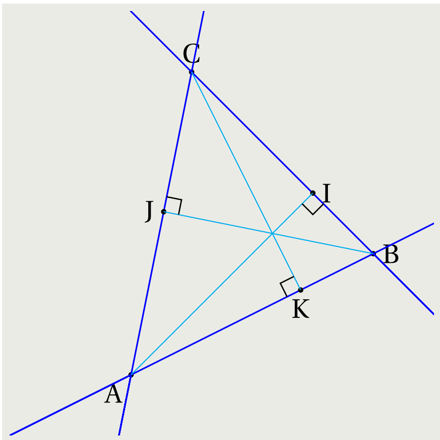
Example n° 78 OrthoCenter




```
\begin{tikzpicture}[scale=.8]
   \tkzInit[xmax=7,ymax=7]\tkzClip
   \SetUpPoint[pos=right]
   \tkzPoint[pos=below left](2,1){A}
   \tkzPoint[pos=right](6,3){B}
   \tkzPoint[pos=above](3,6){C}
   \tkzLine[color=blue](A/B,B/C,A/C)
   \SetUpPoint[pos=left]
   \tkzOrthoCenter[color=red](A,B,C){H}
   \tkzClipPolygon(A,B,C)
   \tkzLine[kl=0,kr=2,color=red](A/H,B/H,C/H)
   \end{tikzpicture}
```

Example n° 79 OrthoCenter

We can use the Projection command to get the altitudes.




```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=7,ymax=7]\tkzClip
  \SetUpPoint[pos=right]
  \tkzPoint[pos=below left](2,1){A}
  \tkzPoint[pos=right](6,3){B}
  \tkzPoint[pos=above](3,6){C}
  \tkzLine[color=blue](A/B,B/C,A/C)
  \SetUpPoint[pos=left]
  \tkzProjection[pos=below](A,B)(C/K)
  \tkzProjection[pos=left](A,C)(B/J)
  \tkzProjection[pos=right](C,B)(A/I)
  \tkzSegment[color=cyan](C/K,A/I,B/J)
  \tkzRightAngle(A/K/C,B/I/A,C/J/B)
\end{tikzpicture}
```

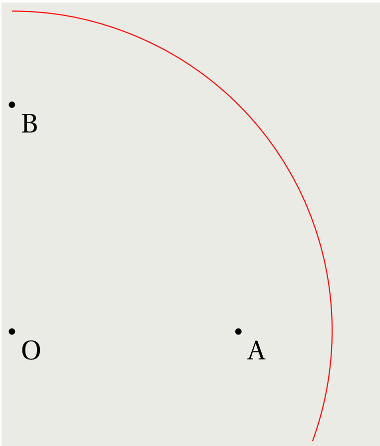
macro n° 41 Arc of unknown radius `\tkzArc`

`\tkzArc[⟨local options⟩](⟨O,r⟩)(⟨V1,V2⟩)`

options	default	definition
color	black	color of the line
lw	0.4pt	width of the line
style	solid	style of the line
out	false	complementary arc

Draw an arc with center O *and a radius* r. V_1 *et* V_2 *are the values of the angles to define the beginning and the end of the arc.*

Example n° 80 Arc of unknown radius with values



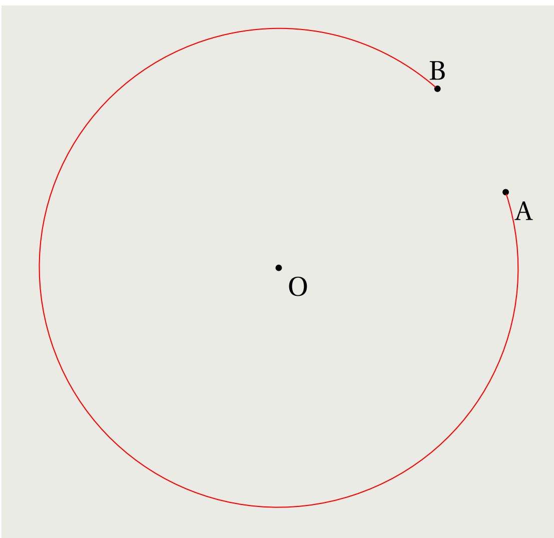

```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint(1,3){O}
  \tkzPoint(4,3){A}
  \tkzPoint(1,6){B}
  \tkzArc[color=red](O,A,B)(-20,90)
\end{tikzpicture}
```

macro n° 42 Arc of unknown radius star-form `\tkzArc*`

`\tkzArc*[⟨local options⟩](⟨O,r⟩)(⟨A,B⟩)`

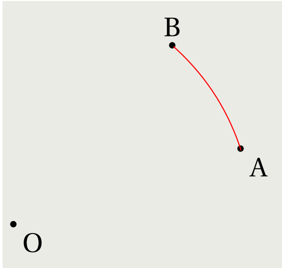
Draw an arc with center O *and a radius* r. A *et* B *are points to define the beginning and the end of the arc.* OA = OB *is necessary.*

Example n° 81 a) Arc of unknown radius with points



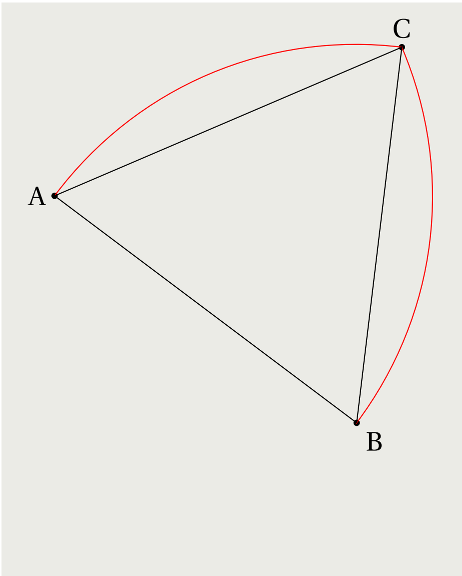

```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint(1,2){O}
  \tkzPoint(4,3){A}
  \tkzRotate[pos=above](O,30)(A/B)
  \tkzArc*[out,color=red](O,O,A)(A,B)
\end{tikzpicture}
```

Example n° 82 b) Arc of unknown radius with points




```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint(1,2){O}
  \tkzPoint(4,3){A}
  \tkzRotate[pos=above](O,30)(A/B)
  \tkzArc*[color=red](O,O,A)(A,B)
\end{tikzpicture}
```

Example n° 83 c) Arc of unknown radius with points



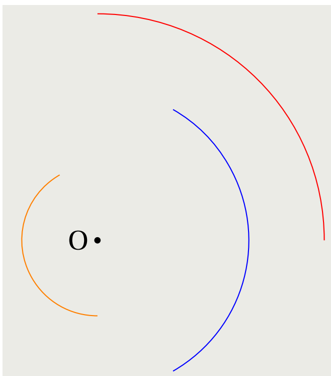

```
\begin{tikzpicture}
  \tkzInit[xmax=7,ymax=7]
  \tkzPoint[pos=left](1,4){A}
  \tkzPoint(5,1){B}
  \tkzSegment(A/B)
  \tkzInterCC(A,A,B)(B,A,B){C}{D}
  \tkzDrawPoint[pos    = above,%
                color = black](C)
  \tkzArc*[color=red,out](A,B,C)(B,C)
  \tkzArc*[color=red](B,A,C)(A,C)
  \tkzSegment(A/C,B/C)
\end{tikzpicture}
```

macro n° 43 Arc of known radius with values`\tkzArcR`

`\tkzArcR[⟨local options⟩](⟨O,r⟩)(⟨V1,V2⟩)`

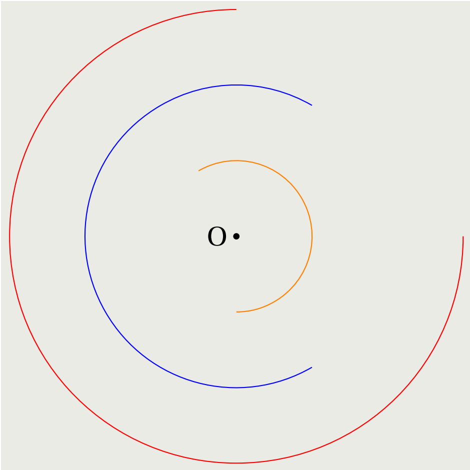
Draw an arc with center O *and radius* r, *the arc begins at* V_1 *and stops at* V_2

Example n° 84 Arc of known radius with values




```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint[pos=left](1,2){O}
  \tkzArcR[color=red](O,3 cm)(0,90)
  \tkzArcR[color=blue](O,2 cm)(-60,60)
  \tkzArcR[color=orange](O,1 cm)(120,270)
\end{tikzpicture}
```

Example n° 85 Arc of known radius with values; `out` option

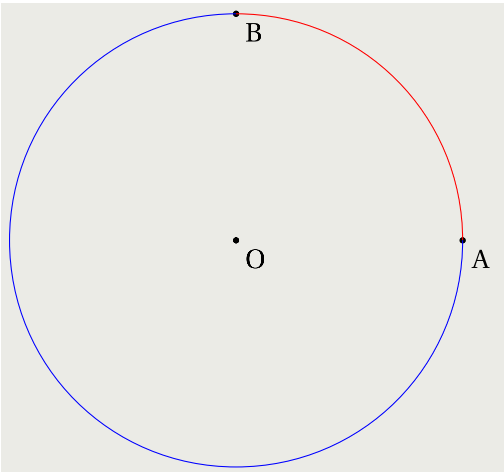



```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint[pos=left](1,2){O}
  \tkzArcR[out,color=red](O,3 cm)(0,90)
  \tkzArcR[out,color=blue](O,2 cm)(-60,60)
  \tkzArcR[out,color=orange](O,1 cm)(120,270)
\end{tikzpicture}
```

macro n° 44 Arc of known radius with points `\tkzArcR*`

`\tkzArcR*[⟨local options⟩](⟨O,r⟩)(⟨A,B⟩)`

Example n° 86 Arc of known radius with points




```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint(1,3){O}
  \tkzPoint(4,3){A}
  \tkzPoint(1,6){B}
  \tkzArcR*[color=red](O,3)(A,B)
  \tkzArcR*[out,color=blue](O,3)(A,B)
\end{tikzpicture}
```

The next commands are useful to indicate marks of a compass.

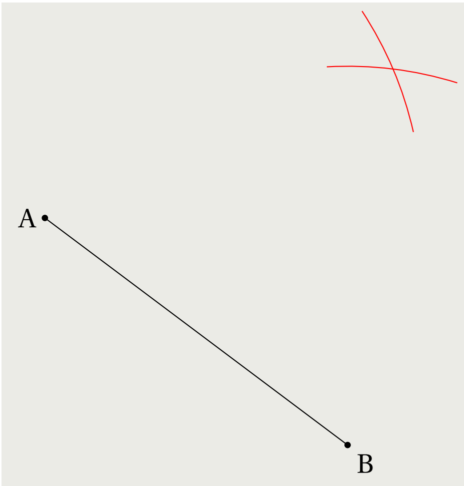
macro n° 45 Compass `\tkzCompass`

`\tkzCompass[⟨local options⟩](⟨O,A⟩)`

options	default	definition
color	black	color of the line
lw	0.4pt	width of the line
style	solid	style of the line
delta	10	length of the arc equal $2 \times$ delta
clock	false	change

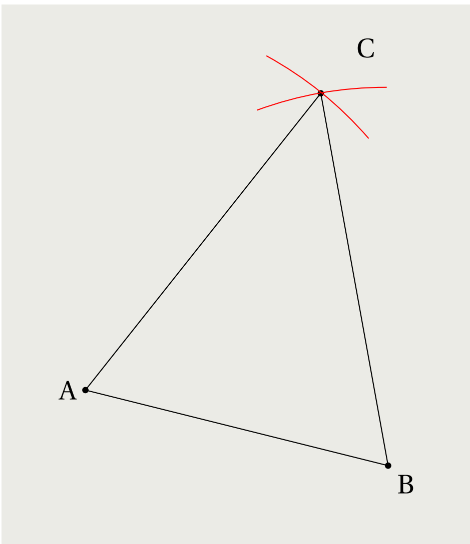
Draw an arc with center A *and radius* r = OA. *The legnth of the arc is given by the option* `delta`. *It's a number of degrees.*

Example n° 87 Compass equilateral triangle



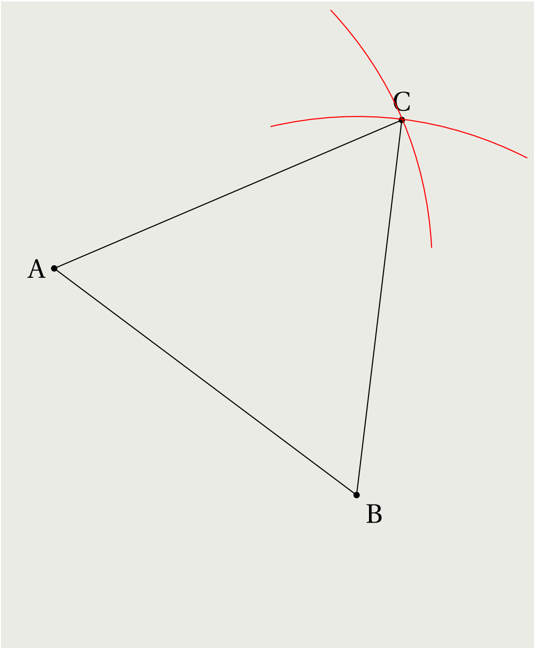

```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint[pos=left](1,5){A}
  \tkzPoint(5,2){B}
  \tkzSegment(A/B)
  \tkzRotate*(A,60)(B/C)
  \tkzCompass[color=red](A,C)
  \tkzCompass[color=red](B,C)
\end{tikzpicture}
```

Example n° 88 Circle-Circle An isosceles triangle.



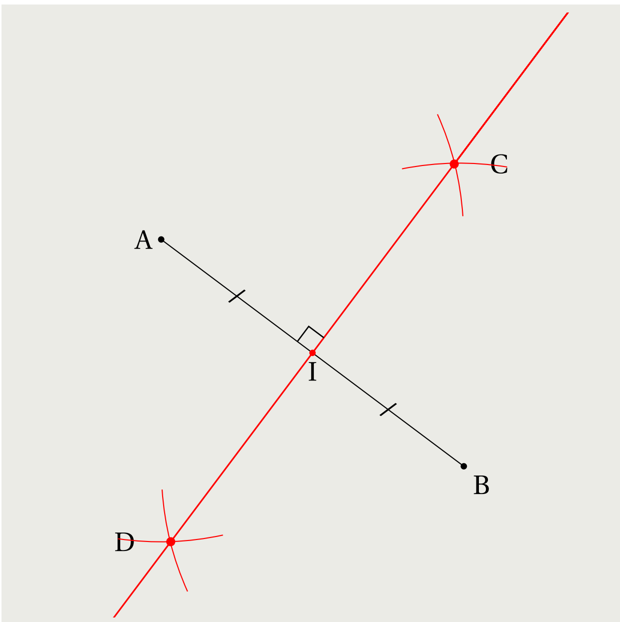

```
\begin{tikzpicture}
  \tkzInit[xmax=6,ymax=7]
  \tkzClip
  \tkzPoint[pos=left](1,2){A}
  \tkzPoint(5,1){B}
  \tkzSegment(A/B)
  \tkzInterCCR(A,5cm)(B,5cm){C}{D}
  \tkzDrawPoint[pos={above right=10pt},%
               color=black](C)
  \tkzCompass[color=red](A,C)
  \tkzCompass[clock,color=red](B,C)
  \tkzSegment(A/C,B/C)
\end{tikzpicture}
```

Example n° 89 Circle-Circle An equilateral triangle.



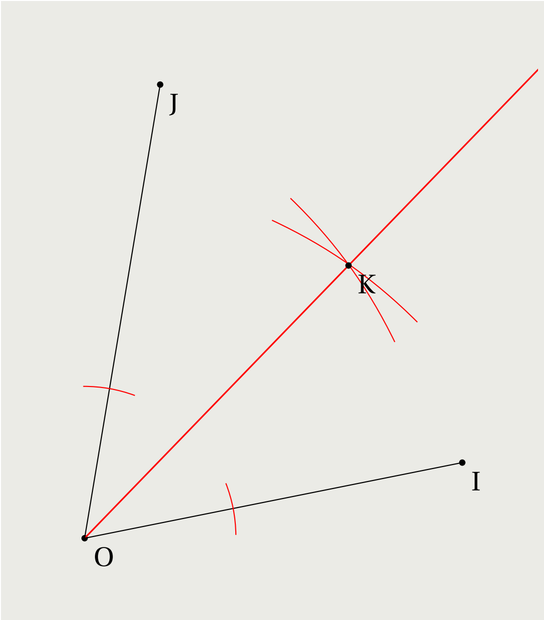

```
\begin{tikzpicture}
  \tkzInit[xmax=7,ymax=7]%\tkzClip
  \tkzPoint[pos=left](1,4){A}
  \tkzPoint(5,1){B}
  \tkzSegment(A/B)
  \tkzInterCC(A,A,B)(B,A,B){C}{D}
  \tkzDrawPoint[pos    = above,%
                color = black](C)
  \tkzCompass[color=red,delta=20](A,C)
  \tkzCompass[color=red,delta=20](B,C)
  \tkzSegment(A/C,B/C)
\end{tikzpicture}
```

Example n° 90 Circle-Circle Mediator Line.




```
\begin{tikzpicture}
  \tkzInit[xmin=-1,xmax=7,ymin=-1,ymax=7]\tkzClip
  \tkzPoint[pos=left](1,4){A} \tkzPoint(5,1){B}
  \tkzSegment(A/B)\tkzMidPoint[color=red,pos=below](A,B){I}
  \tkzInterCCR(A,4 cm)(B,4 cm){C}{D}
  \tkzDrawPoint[pos={right=10pt},color=red,size=1.5pt](C)
  \tkzDrawPoint[pos={left=10pt},color=red,size=1.5pt](D)
  \tkzCompass[color=red](A,C)
  \tkzCompass[clock,color=red](B,C)
  \tkzCompass[color=red](A,D)
  \tkzCompass[clock,color=red](B,D)
  \tkzLine[color=red](C/D)
  \tkzRightAngle(A/I/C)
  \tkzSegmentMark[symbol=/](A/I,B/I)
\end{tikzpicture}
```

Example n° 91 Bisector.



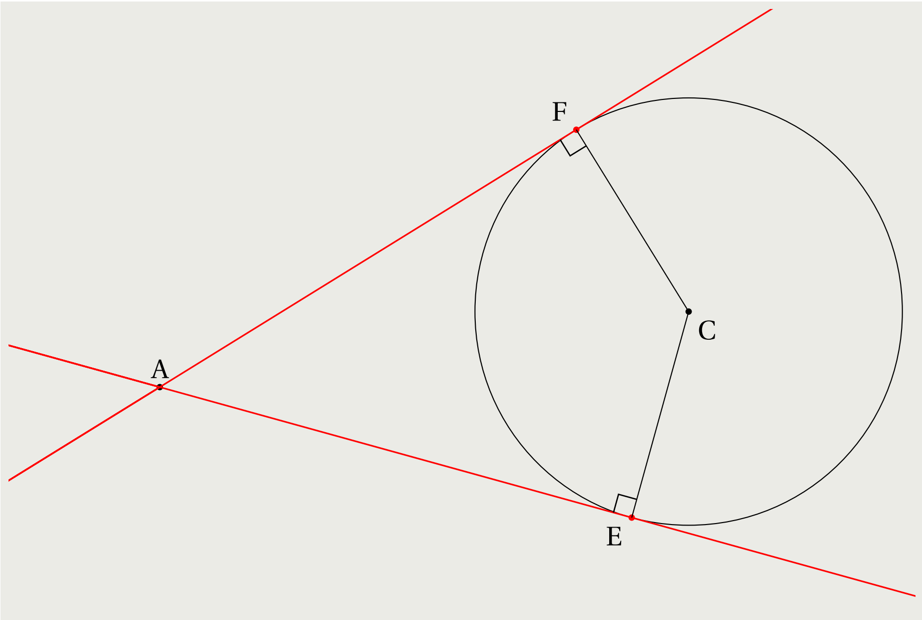

```
\begin{tikzpicture}
  \tkzInit[xmin=-1,xmax=6,ymin=-1,ymax=7]
  \tkzClip
  \tkzPoint(0,0){O}
  \tkzPoint(5,1){I}
  \tkzPoint(1,6){J}
  \tkzBisector[color=red,kl=0](I,O,J){K}
  \tkzInterLCR(O,I)(O,2cm){C}{D}
  \tkzInterLCR(O,J)(O,2cm){E}{F}
  \tkzSegment(O/I,O/J)
  \tkzCompass[color=red](O,C)
  \tkzCompass[color=red](O,E)
  \tkzCompass[color=red](C,K)
  \tkzCompass[color=red](E,K)
  \tkzDrawPoint(K)
\end{tikzpicture}
```

macro n° 46 Tangent to an circle with a known radius `\tkzTgtFromPR`

`\tkzTgtFromPR[⟨local options⟩](⟨O,r⟩)(⟨A⟩){⟨E⟩}(⟨F⟩)`

A is a point outside a circle of center O and radius r. We get the two points E and F where lines from A are tangent at the circle.

Example n° 92 a) Tangent to a circle from a point



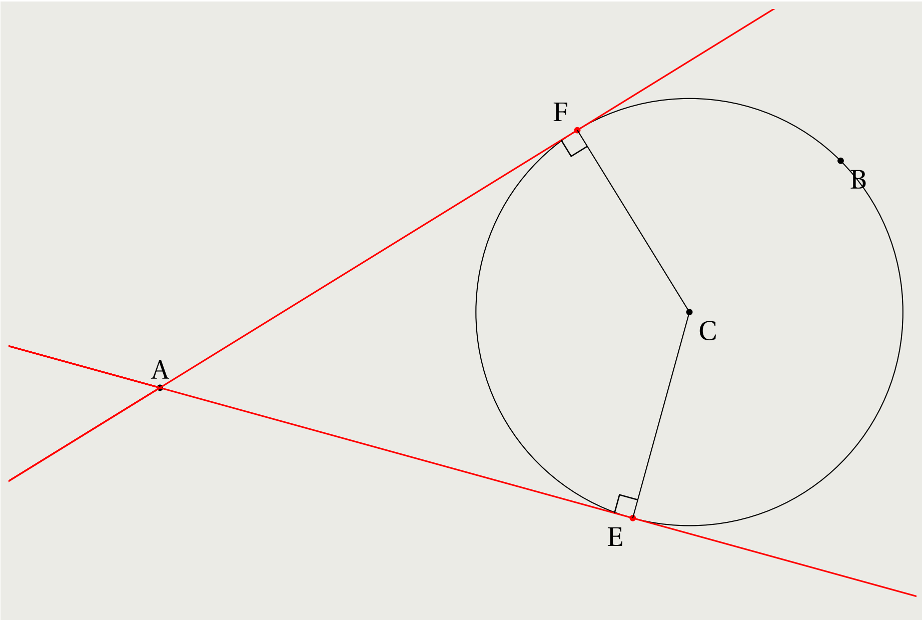

```
\begin{tikzpicture}
  \tkzInit[xmin=-6,xmax=6,ymin=-1,ymax=7]
  \tkzClip
  \tkzPoint(3,3){C}
  \tkzPoint[pos=above](-4,2){A}
  \pgfmathparse{2*sqrt(2)}
  \let\tempresult\pgfmathresult
  \tkzCircleR(C,\tempresult cm)
  \tkzTgtFromPR(C,\tempresult cm)(A){E}{F}
  \tkzDrawPoint[pos=below left,color=red](E)
  \tkzDrawPoint[pos=above left,color=red](F)
  \tkzLine[color=red](A/E,A/F)
  \tkzRightAngle(A/E/C,A/F/C)
  \tkzSegment(C/E,C/F)
\end{tikzpicture}
```

macro n° 47 Tangent to an circle with a unknown radius `\tkzTgtFromP`

`\tkzTgtFromPR[⟨local options⟩](⟨O,I,J⟩)(⟨A⟩){⟨E⟩}(⟨F⟩)`

A *is a point outside a circle of center* O *and radius* r = IJ. *We get the two points* E *and* F *where lines from* A *are tangent at the circle.*

Example n° 93 Tangent to a circle from a point n°2



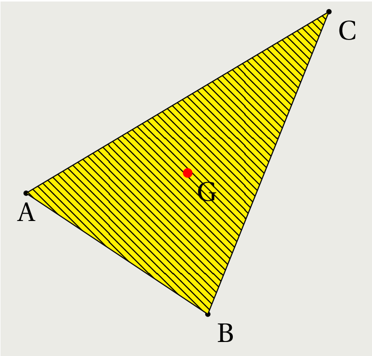

```
\begin{tikzpicture}[scale=1]
  \tkzInit[xmin=-6,xmax=6,ymin=-1,ymax=7]
  \tkzClip
  \tkzPoint(3,3){C}
  \tkzPoint(5,5){B}
  \tkzPoint[pos=above](-4,2){A}
  \tkzCircleR*(C,C,B)
  \tkzTgtFromP(C,C,B)(A){E}{F}
  \tkzDrawPoint[pos=below left,color=red](E)
  \tkzDrawPoint[pos=above left,color=red](F)
  \tkzLine[color=red](A/E,A/F)
  \tkzRightAngle(A/E/C,A/F/C)
  \tkzSegment(C/E,C/F)
\end{tikzpicture}
```

macro n° 48 Fill Polygon `\tkzFillPolygon`

`\tkzFillPolygon[⟨local options⟩](⟨ A,B,...⟩)`

The local options is the same for all the commands about fill

Example n° 94 Fill Polygon

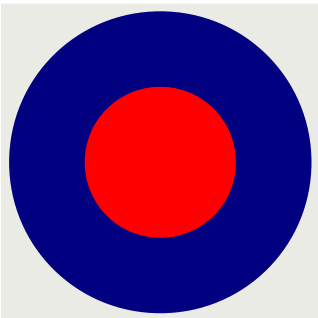



```
\begin{tikzpicture}[scale=.8]
   \tkzInit
   \tkzPoint[pos=below](1,3){A}
   \tkzPoint(4,1){B}
   \tkzPoint(6,6){C}
   \tkzGravityCenter*(A,B,C){G}
   \tkzFillPolygon[color=yellow](A,B,C)
   \tkzFillPolygon[pattern={north west lines}](A,B,C)
   \tkzPolygon(A,B,C)
   \tkzDrawPoint[color=red,size=2pt](G)
\end{tikzpicture}
```

macro n° 49 Fill Circle `\tkzFillCircle`

`\tkzFillCircle[⟨local options⟩](⟨A,r⟩)`

Example n° 95 Fill Circle

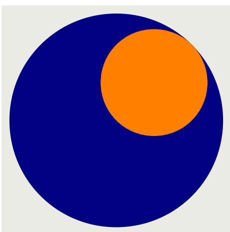


```
\begin{tikzpicture}
   \tkzInit[xmax=5,ymax=5]
   \tkzPoint*(3,3){B}
   \tkzFillCircle[color=blue!50!black](B,2 cm)
   \tkzFillCircle[color=red](B,1 cm)
\end{tikzpicture}
```

macro n° 50 Fill Circle with unknown radius `\tkzFillCircle*`

`\tkzFillCircle*[⟨local options⟩](⟨A,E,F⟩)`

Example n° 96 Fill Circle with unknown radius



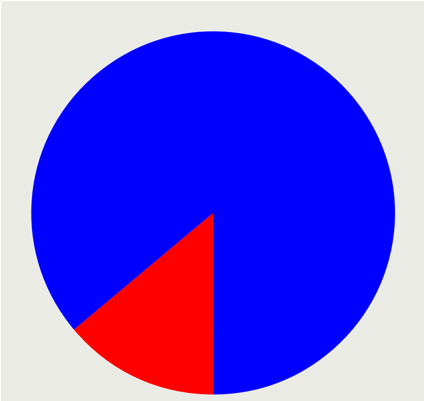
```
\begin{tikzpicture}
   \tkzInit[xmax=5,ymax=5]
   \tkzPoint*(2.5,2.5){A}
   \tkzPoint*(2,2){E}
   \tkzPoint*(3,3){F}
   \tkzFillCircle*[color=blue!50!black](A,E,F)
   \tkzFillCircle*[color=orange](F,F,A)
\end{tikzpicture}
```

macro n° 51 Fill Sector `\tkzFillSector`

`\tkzFillSector[⟨local options⟩](⟨A,r⟩)(⟨V1,V2⟩)`

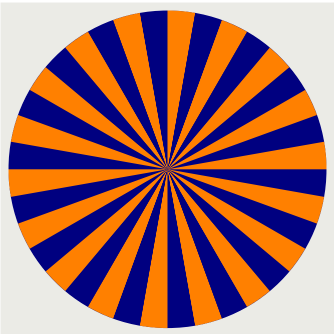
It is possible to fill a sector with a circle (center;radius) and two values in degrees

Example n° 97 a- Fill Sector



```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=8,ymax=8]
  \tkzPoint(4,4){A}
  \tkzCircleR(A,2 cm)
  \tkzFillSector[color=blue,out](A,3 cm)(220,270)
  \tkzFillSector[color=red](A,3 cm)(220,270)
\end{tikzpicture}
```

Example n° 98 b- Fill Sector



```
\begin{tikzpicture}[scale=.7]
  \tkzInit[xmax=8,ymax=8]
  \tkzPoint*(4,4){B}
  \tkzFillCircle[color=blue!50!black](B,3 cm)
  \foreach \ang in {0,20,...,340}{%
   \edef\angmore{\ang+10}
   \tkzFillSector[color=orange](B,3 cm)(\ang,\angmore)}
\end{tikzpicture}
```

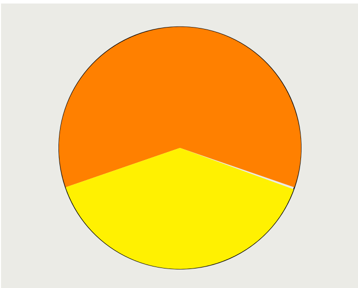
macro n° 52 Fill Sector star-form `\tkzFillSector*`

`\tkzFillSector*[⟨local options⟩](⟨A,r⟩)(⟨V1,V2⟩)`

It is possible to fill a sector with a circle (center;radius) and two values in degrees

Example n° 99 Fill Sector star-form

The result is not perfect because the arc macro is buggy when we use decimal degrees.



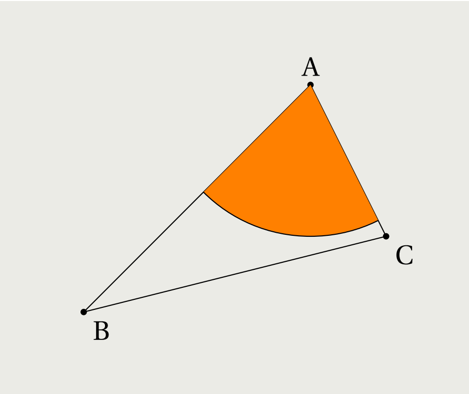
```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=8,ymax=8]
  \tkzPoint*(4,4){A}
  \tkzCircleR(A,2 cm)
  \tkzPoint*(2,3){B}
  \tkzPoint*(4,3){C}
  \tkzInterLCR(B,C)(A,3 cm){I}{J}
  \tkzFillSector*[color=orange,out](A,2 cm)(I,J)
  \tkzFillSector*[color=yellow](A,2 cm)(I,J)
\end{tikzpicture}
```

macro n° 53 Clip Polygon `\tkzClipPolygon`

`\tkzClipPolygon[⟨local options⟩](⟨A,B,...⟩)`

This command define a polygon as a clipping aera.

Example n° 100 Clip Polygon



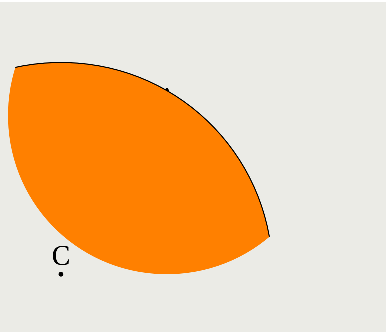

```
\begin{tikzpicture}
   \tkzInit[xmax=6,ymax=5]\tkzClip
   \tkzPoint[pos=above](4,4){A}
   \tkzPoint(1,1){B}
   \tkzPoint(5,2){C}
   \tkzPolygon(A,B,C)
   \tkzClipPolygon(A,B,C)
   \tkzCircleR[style={fill=orange}](A,2 cm)
\end{tikzpicture}
```

macro n° 54 Clip Circle `\tkzClipCircle`

`\tkzClipCircle[⟨local options⟩](⟨O,r⟩)`

This command define a circle as a clipping aera. You need to give the center O *and the radius* r.

Example n° 101 Clip Circle



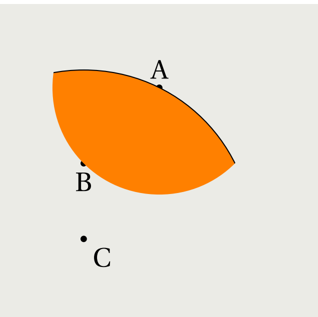

```
\begin{tikzpicture}[scale=.7]
  \tkzInit[xmax=7,ymax=6]\tkzClip
  \tkzPoint[pos=above](3,4){A}
  \tkzPoint[pos=above](1,1){C}
  \tkzClipCircle(A,3 cm)
  \tkzCircleR[style={fill=orange}](C,4 cm)
\end{tikzpicture}
```

macro n° 55 Clip Circle star-form `\tkzClipCircle*`

`\tkzClipCircle*[⟨local options⟩](⟨O,A,B⟩)`

This command define a circle as a clipping aera. With the star-form, you need to define the radius with the length of the segment AB, O *is the center.*

Example n° 102 Clip Circle star-form



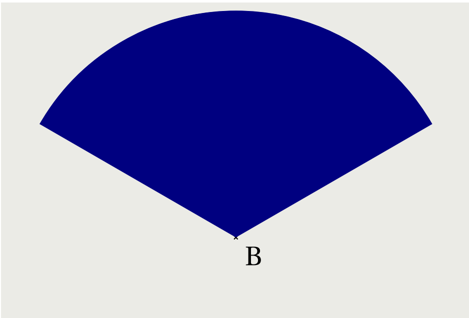

```
\begin{tikzpicture}[scale=1]
  \tkzInit[xmin=2,xmax=6,ymin=2,ymax=6]\tkzClip
  \tkzPoint[pos=above](4,5){A}
  \tkzPoint[pos=below](3,4){B}
  \tkzPoint(3,3){C}
  \tkzClipCircle*(A,A,B)
  \tkzCircleR*[style={fill=orange}](C,C,A)
\end{tikzpicture}
```

macro n° 56 Clip Sector `\tkzClipSector`

`\tkzClipSector[⟨local options⟩](⟨O,r⟩)(⟨V1,V2⟩)`

This command define a sector as a clipping aera. Thus, if you want only a local effect of the `\clip` *command, you need to use it somewhere inside a scope.*

Example n° 103 Clip Sector




```
\begin{tikzpicture}
   \tkzInit[xmax=6,ymax=4] \tkzClip
   \tkzPoint[mark=x](3,1){B}
   \tkzClipSector(B,3 cm)(30,150)
   \tkzFillCircle[color=blue!50!black](B,3 cm)
\end{tikzpicture}
```

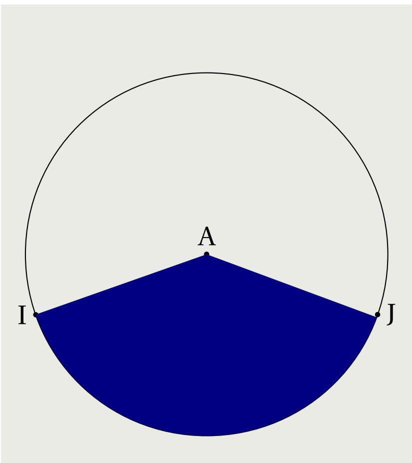
macro n° 57 Clip Sector star-form `\tkzClipSector*`

`\tkzClipSector*[⟨local options⟩](⟨O,r⟩)(⟨A,B⟩)`

The star-form of the last command is usefull if you want to use point instead of values of angles.

Example n° 104 ClipSector star-form

The rsult is not perfect (see the point J)




```
\begin{tikzpicture}[scale=.8]
  \tkzInit[xmax=8,ymax=8]
  \tkzPoint[pos=above](4,4){A}
  \tkzCircleR(A,3 cm)
  \tkzPoint*(2,3){B}
  \tkzPoint*(6,3){C}
  \tkzInterLCR(B,C)(A,3 cm){I}{J}
  \tkzDrawPoint[pos=left](I)
  \tkzDrawPoint[pos=right](J)
  \tkzClipSector*(A,3 cm)(I,J)
  \tkzFillCircle[color=blue!50!black](A,3 cm)
\end{tikzpicture}
```

VIII . Intersections

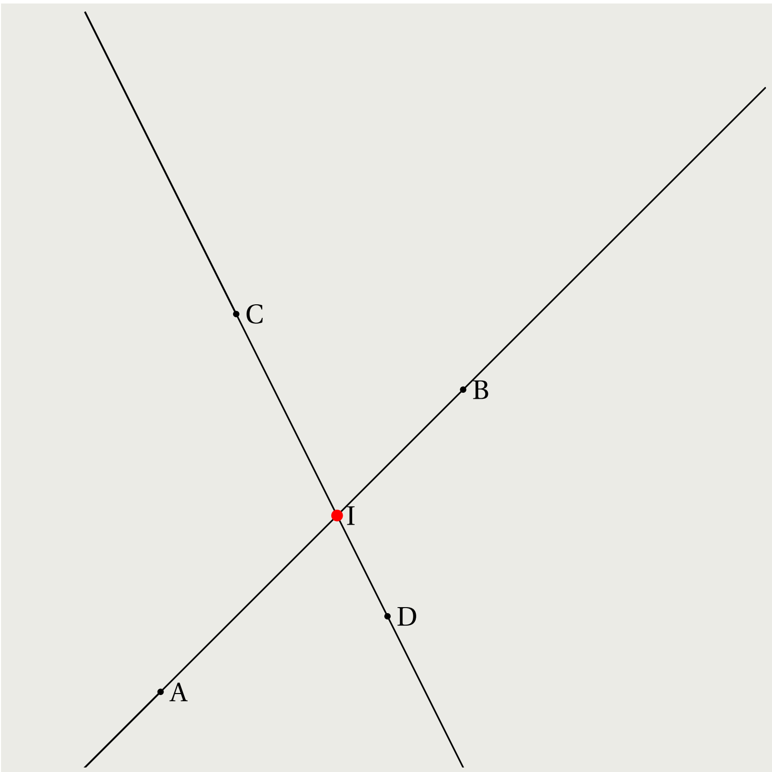
The user can define points by intersection, three types of intersections are managed:
- Intersection point defined by two different lines,
- Intersection point defined by one line and one circle,
- Intersection point defined by two different circles.

Each command has a associated set-form. The local options (optionnal arguments) for these macros are the point options. The user has to manage the existence of the intersection point.

macro n° 58 Intersection Line-Line `\tkzInterLL`

`\tkzInterLL[⟨Point options⟩](⟨A,B⟩)(⟨C,D⟩){⟨I⟩}`
Draw or define (star-form) the intersection point I *of two lines* (AB) *and* (CD) *with point options.*

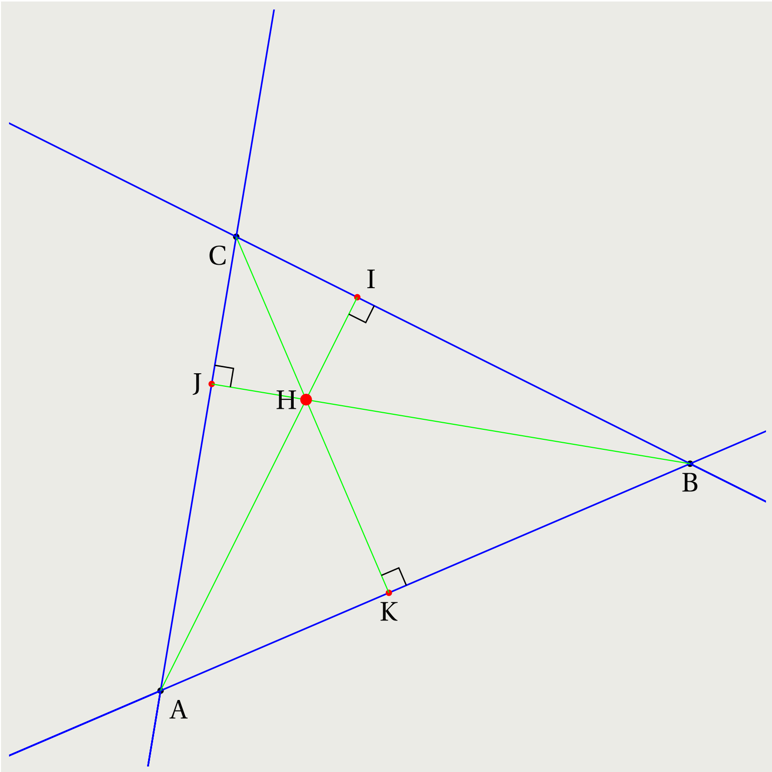
Example n° 105 Line-Line




```
\begin{tikzpicture}
    \tkzInit\tkzClip
    \SetUpPoint[pos=right]
    \tkzPoints(2/1/A,%
               6/5/B,%
               3/6/C,%
               5/2/D)
    \tkzLine(A/B,C/D)
    \tkzInterLL[size=2pt,color=red](A,B)(C,D){I}
 \end{tikzpicture}
```

Example n° 106 Line-Line OrthoCenter

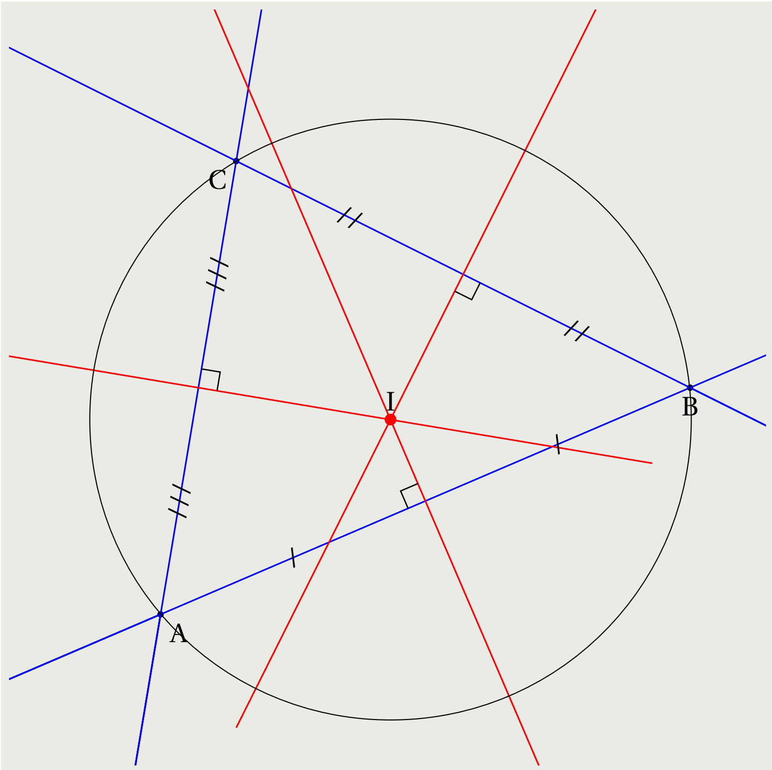
The intersection of the three altitudes of a triangle is called the orthocenter.




```
\begin{tikzpicture}
    \tkzInit\tkzClip
    \tkzPoint[pos=below right](2,1){A}%
    \tkzPoint[pos=below](9,4){B}%
    \tkzPoint[pos=below left](3,7){C}%
    \tkzLine[color=blue](A/B,B/C,A/C)
    \SetUpPoint[pos=left]
    \tkzProjection[color=red,pos=below](A,B)(C/K)
    \tkzProjection[color=red](A,C)(B/J)
    \tkzProjection[color=red,pos=above right](B,C)(A/I)
    \tkzSegment[color=green](A/I,B/J,C/K)
    \tkzInterLL[color=red,size=2pt](A,I)(B,J){H}
    \tkzRightAngle(A/I/B,B/J/C,C/K/B)
\end{tikzpicture}
```

Example n° 107 Line-Line CircumCenter

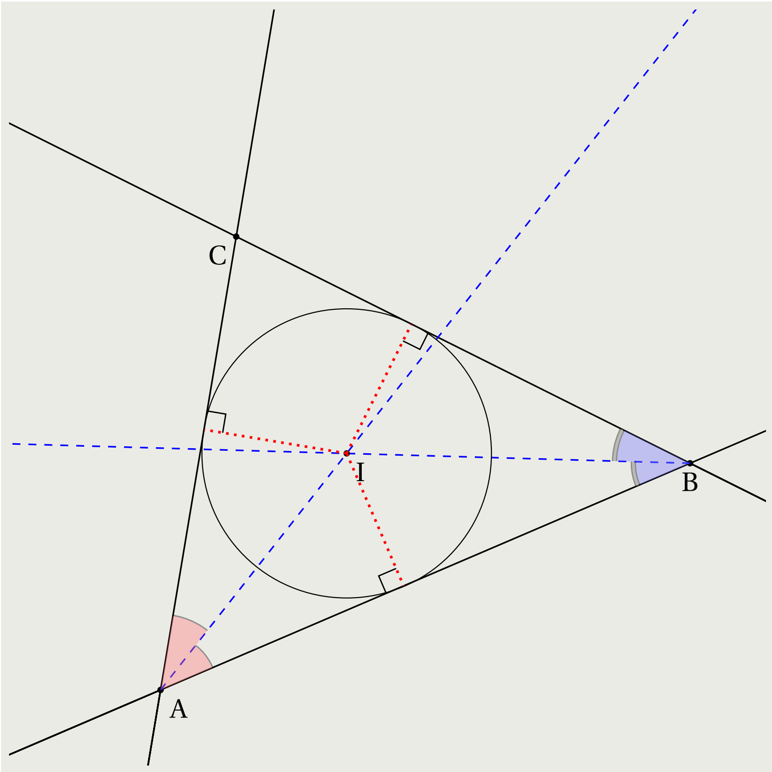
The circumcenter can be found as the intersection of the perpendicular bisectors (mediator-line), is the center of a triangle's circumcircle.




```
\begin{tikzpicture}
    \tkzInit[ymin=-1,ymax=9]\tkzClip
    \tkzPoint[pos=below right](2,1){A}%
    \tkzPoint[pos=below](9,4){B}%
    \tkzPoint[pos=below left](3,7){C}%
    \tkzLine[color=blue](A/B,B/C,A/C)
    \SetUpPoint[pos=left]
    \tkzMediatorLine[prefix=m1,color=red](A,B)
    \tkzMediatorLine[prefix=m2,color=red](B,C)
    \tkzMediatorLine[color=red](A,C){B}
    \tkzInterLL*(m1l,m1r)(m2r,m2l){I}
    \tkzDrawPoint[color=red,size=2pt,pos=above](I)
    \tkzProjection*(A,B)(I/c)
    \tkzProjection*(A,C)(I/b)
    \tkzProjection*(B,C)(I/a)
    \tkzRightAngle(A/c/I,B/a/I,C/b/I)
    \tkzSegmentMark[symbol=/](A/c,B/c)
    \tkzSegmentMark[symbol=//](B/a,C/a)
    \tkzSegmentMark[symbol=///](C/b,A/b)
    \tkzCircle(I,A)
\end{tikzpicture}
```

Example n° 108 Line-Line InCenter

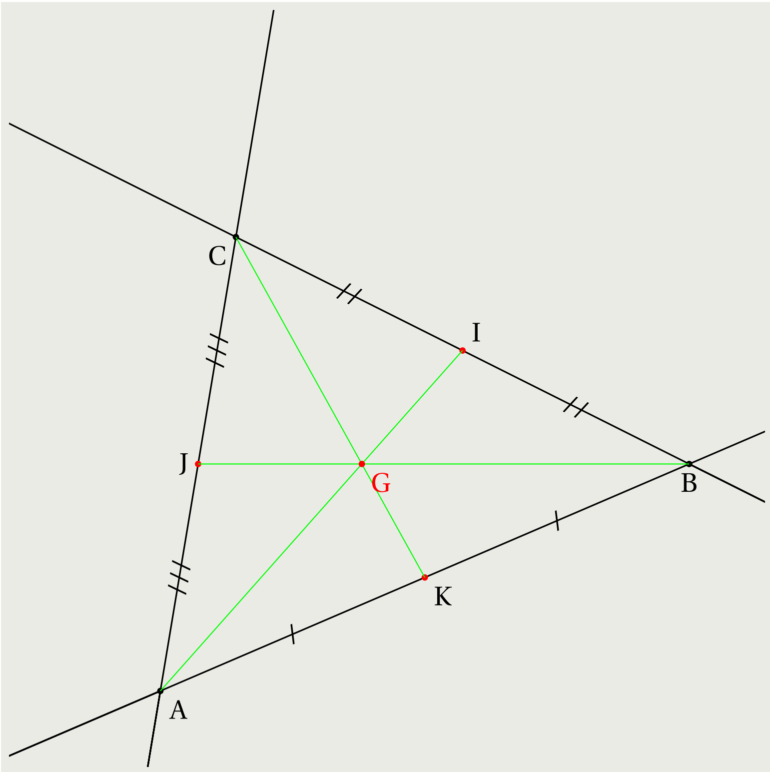
The incenter I is the center of the incircle for a given triangle. It can be found as the intersection of angle bisectors, and it is the interior point for which distances to the sides of the triangle are equal.




```
  \begin{tikzpicture}
      \tkzInit\tkzClip
      \tkzPoint[pos=below right](2,1){A}%
      \tkzPoint[pos=below](9,4){B}%
      \tkzPoint[pos=below left](3,7){C}%
      \tkzLine(A/B,B/C,A/C)
      \tkzBisector[kl=0,kr=3,color=blue,style=dashed](B,A,C){x}
      \tkzBisector[kl=0,kr=3,color=blue,style=dashed](A,B,C){y}
      \tkzInterLL(A,x)(B,y){I}
      \tkzProjection*(A,B)(I/c)
      \tkzProjection*(A,C)(I/b)
      \tkzProjection*(B,C)(I/a)
      \tkzSegment[color=red,style=dotted,lw=1pt](I/a,I/b,I/c)
      \tkzRightAngle(A/c/I,B/a/I,C/b/I)
      \tkzMarkAngle[size      = 1,%
                    style     = ai,%
                    fillcolor = red!50](I/A/C)
      \tkzMarkAngle[size      = 0.75,%
                    style     = ai,%
                    fillcolor = red!50](I/A/B)
      \tkzMarkAngle[size      = 1,%
                    style     = aii,%
                    fillcolor = blue!50](I/B/C)
      \tkzMarkAngle[size      = 0.75,%
                    style     = aii,%
                    fillcolor = blue!50](I/B/A)
      \tkzCircle(I,a)
 \end{tikzpicture}
```

Example n° 109 Line-Line GravityCenter

A median of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas. The three medians intersect in a single point, the triangle's center of gravity.



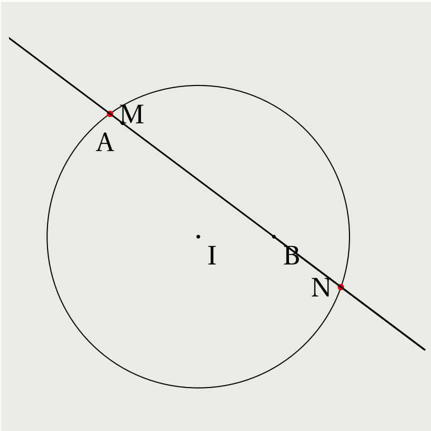

```
\begin{tikzpicture}
    \tkzInit\tkzClip
    \tkzPoint[pos=below right](2,1){A}%
    \tkzPoint[pos=below](9,4){B}%
    \tkzPoint[pos=below left](3,7){C}%
    \tkzLine(A/B,B/C,A/C)
    \tkzMidPoint[color=red](A,B){K}
    \tkzMidPoint[color=red,pos=above right](B,C){I}
    \tkzMidPoint[color=red,pos=left](A,C){J}
    \tkzSegment[color=green](A/I,B/J,C/K)
    \tkzInterLL[color=red,namecolor=red](A,I)(B,J){G}
    \tkzSegmentMark[symbol=/](A/K,B/K)
    \tkzSegmentMark[symbol=//](B/I,C/I)
    \tkzSegmentMark[symbol=///](C/J,A/J)
\end{tikzpicture}
```

macro n° 59 Intersection Line-Circle with a known radius `\tkzInterLCR`

`\tkzInterLCR[⟨Point options⟩](⟨A,B⟩)(⟨I,r⟩){⟨M⟩}{⟨N⟩}`

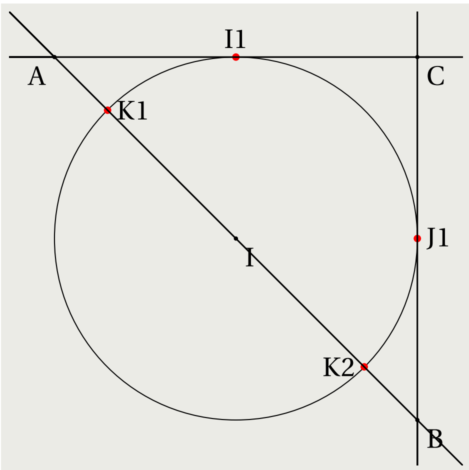
Determine the intersection (M *and* N*) of a line* (AB) *and a circle with a center* I *and a radius* r.

Example n° 110 Line-Circle general case



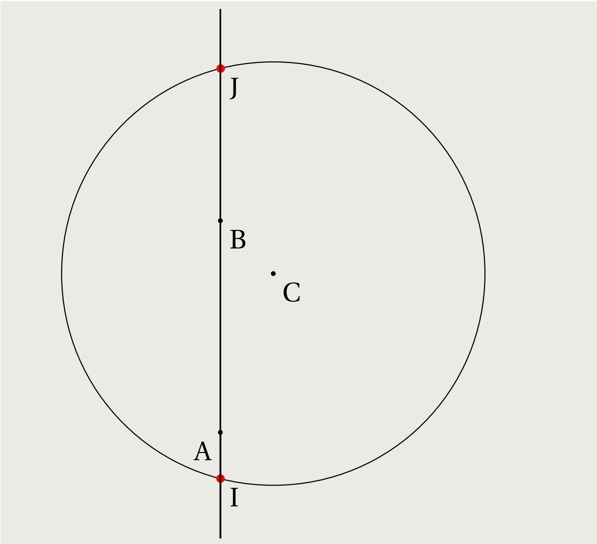

```
\begin{tikzpicture}[scale=.5]
  \tkzInit[xmin=-1,xmax=10,ymin=-1,ymax=10]
  \tkzClip
  \tkzPoint[pos=below left](2,7){A}
  \tkzPoint(6,4){B}\tkzPoint(4,4){I}
  \tkzCircleR(I,4 cm)
  \tkzInterLCR(A,B)(I,4 cm){M}{N}
  \tkzDrawPoint[size=2pt,color=red,pos=right](M)
  \tkzDrawPoint[size=2pt,color=red,pos=left](N)
  \tkzLine(B/A)
\end{tikzpicture}
```

Example n° 111 Line-Circle center on the line



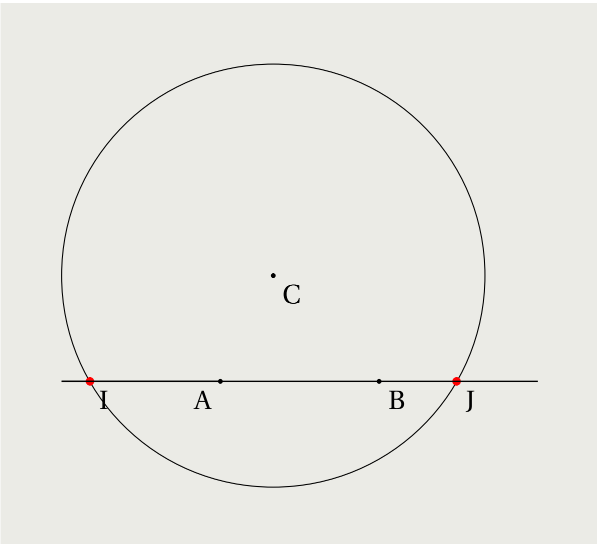

```
\begin{tikzpicture}[scale=.6]
  \tkzInit[xmin=-1,xmax=9,%
           ymin=-1,ymax=9]
  \tkzClip
  \tkzPoint[pos=below left](0,8){A}
  \tkzPoint(8,0){B}
  \tkzPoint(8,8){C}
  \tkzPoint(4,4){I}
  \tkzCircleR(I,4 cm)
  \tkzInterLCR(A,C)(I,4 cm){I1}{I2}
  \tkzInterLCR(B,C)(I,4 cm){J1}{J2}
  \tkzInterLCR(A,B)(I,4 cm){K1}{K2}
  \tkzDrawPoint[mark=*,size=2pt,%
                color=red,pos=above](I1)
  \tkzDrawPoint[mark=*,size=2pt,%
                color=red,pos=right](J1,K1)
  \tkzDrawPoint[mark=*,size=2pt,%
                color=red,pos=left](K2)
  \tkzLine(A/B,B/C,A/C)
\end{tikzpicture}
```

Example n° 112 Line-Circle vertical line




```
\begin{tikzpicture}[scale=.7]
  \tkzInit[xmin=-2,xmax=9,%
           ymin=-1,ymax=9]
  \tkzClip
  \tkzPoint[pos=below left](2,1){A}
  \tkzPoint(2,5){B}
  \tkzPoint(3,4){C}
  \tkzCircleR(C,4 cm)
  \tkzInterLCR(A,B)(C,4 cm){I}{J}
  \tkzDrawPoint[size=2pt,color=red](I,J)
  \tkzLine(A/B)
\end{tikzpicture}
```

Example n° 113 Line-Circle horizontal line



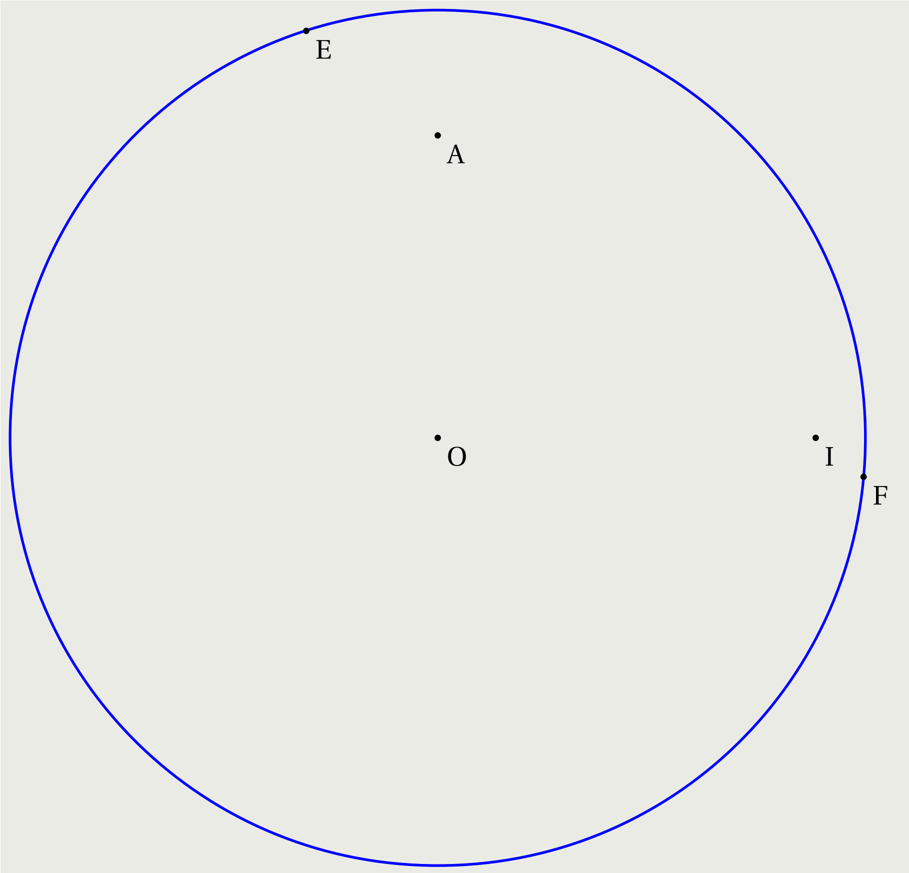

```
\begin{tikzpicture}[scale=.7]
  \tkzInit[xmin=-2,xmax=9,%
           ymin=-1,ymax=9]
  \tkzClip
  \tkzPoint[pos=below left](2,2){A}
  \tkzPoint(5,2){B}
  \tkzPoint(3,4){C}
  \tkzCircleR(C,4 cm)
  \tkzInterLCR(A,B)(C,4 cm){I}{J}
  \tkzDrawPoint[size=2pt,color=red](I,J)
  \tkzLine(A/B)
 \end{tikzpicture}
```

macro n° 60 Intersection Line-Circle with points `\tkzInterLC`

`\tkzInterLC[⟨Point options⟩](⟨A,B⟩)(⟨O,C,D⟩){⟨M⟩}{⟨N⟩}`

Draw or define (star-form) the intersection (M *and* N*) of a line* (AB) *and a circle with a center* I *and a radius given by* CD.

Example n° 114 Line-Circle with points

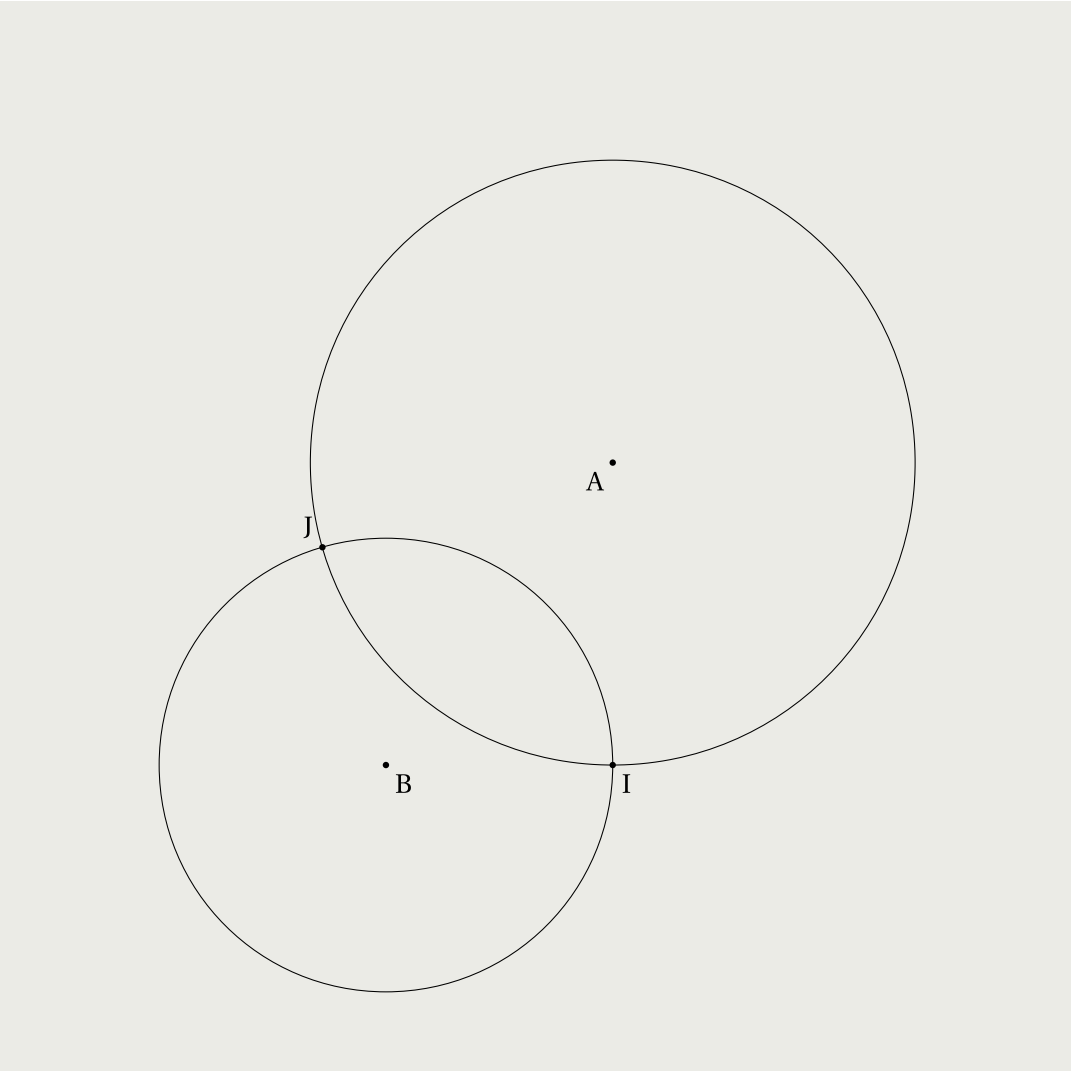



```
\begin{tikzpicture}
\tkzInit
\tkzPoint(0,0){O} \tkzPoint(0,4){A} \tkzPoint(5,0){I}
\tkzPoint*(4,4){K}
\tkzCircleR*[color=blue,lw=1pt](O,O,K)
\tkzInterLC(A,I)(O,O,K){E}{F}
\tkzDrawPoint(E,F)
\end{tikzpicture}
```

macro n° 61 Intersection Circle-Circle `\tkzInterCCR`

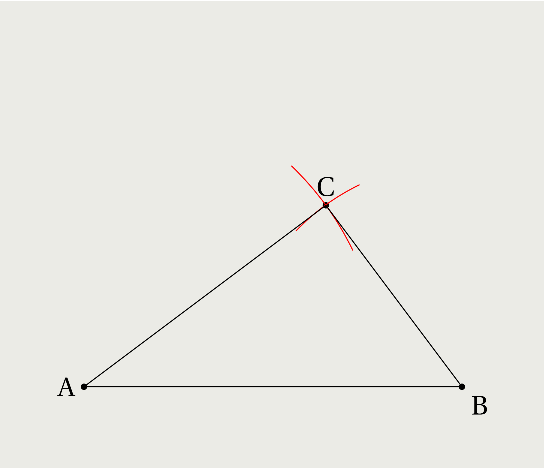
`\tkzInterCCR[⟨Point options⟩](⟨A, r1⟩)(⟨B, r2⟩){⟨M⟩}{⟨N⟩}` *Determine the intersection (* M *and* N*) of two circles with respectively, centers* A *and* B*, radii* $r1$ *and* $r2$.

Example n° 115 Circle-Circle General case



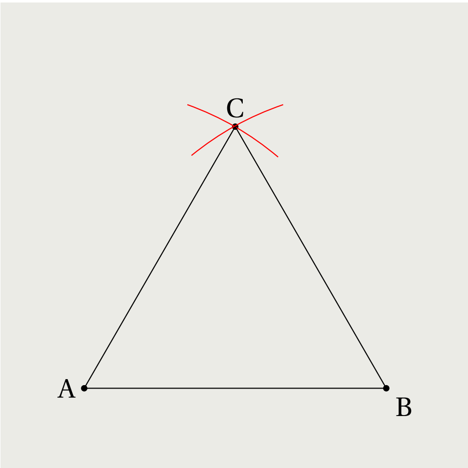

```
\begin{tikzpicture}
  \tkzInit[xmin=-2,xmax=12,%
           ymin=-2,ymax=12]
  \tkzClip
  \tkzPoint[pos=below left](6,6){A}
  \tkzPoint(3,2){B}
  \tkzCircleR(A,4 cm)
  \tkzCircleR(B,3 cm)
  \tkzInterCCR(A,4 cm)(B,3 cm){I}{J}
  \tkzDrawPoint(I)
  \tkzDrawPoint[pos=above left](J)
\end{tikzpicture}
```

Example n° 116 Circle-Circle Compass



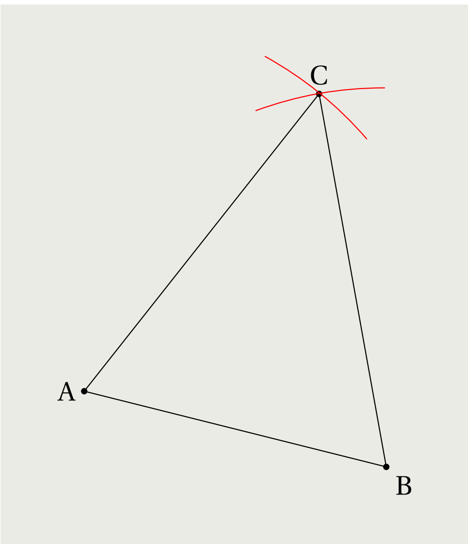

```
\begin{tikzpicture}
    \tkzInit[xmax=7,ymax=6]
    \tkzClip
    \tkzPoint[pos=left](1,1){A}
    \tkzPoint(6,1){B}
    \tkzInterCCR(A,4cm)(B,3cm){C}{D}
    \tkzDrawPoint[pos=above,%
                  color=black](C)
    \tkzCompass[color=red](A,C)
    \tkzCompass[clock,%
                color=red](B,C)
    \tkzSegment(A/B,A/C,B/C)
\end{tikzpicture}
```

Example n° 117 Circle-Circle Compass




```
\begin{tikzpicture}
 \tkzInit[xmax=6,ymax=6]
 \tkzClip
 \tkzPoint[pos=left](1,1){A}
 \tkzPoint(5,1){B}
 \tkzInterCCR(A,4cm)(B,4cm){C}{D}
 \tkzDrawPoint[pos=above,%
               color=black](C)
 \tkzCompass[color=red](A,C)
 \tkzCompass[clock,%
             color=red](B,C)
 \tkzSegment(A/B,A/C,B/C)
\end{tikzpicture}
```

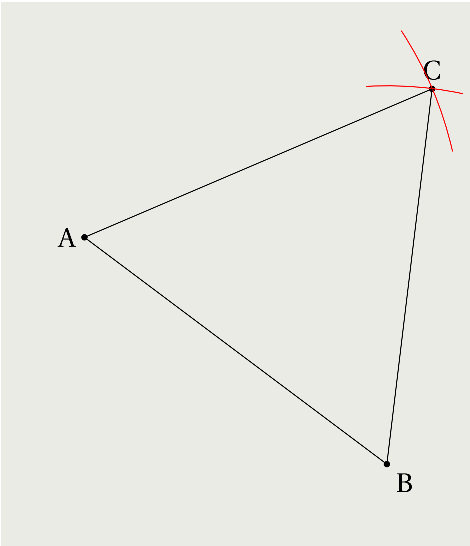
Example n° 118 Circle-Circle An isosceles triangle.




```
\begin{tikzpicture}
 \tkzInit[xmax=6,ymax=7]\tkzClip
 \tkzPoint[pos=left](1,2){A}
 \tkzPoint(5,1){B}
 \tkzInterCCR(A,5cm)(B,5cm){C}{D}
 \tkzDrawPoint[pos=above,%
               color=black](C)
 \tkzCompass[color=red](A,C)
 \tkzCompass[clock,%
             color=red](B,C)
 \tkzSegment(A/B,A/C,B/C)
\end{tikzpicture}
```

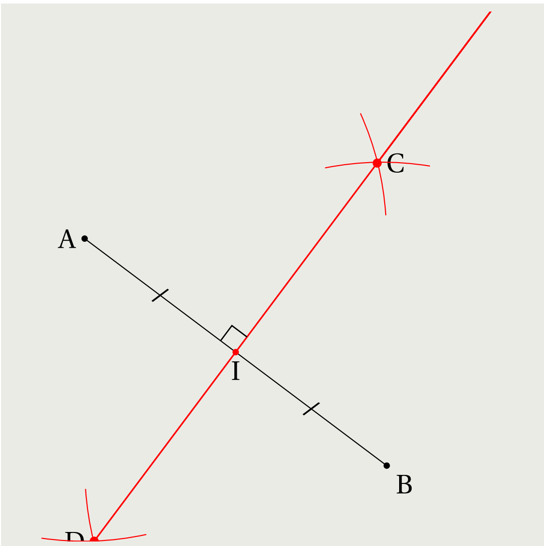
Example n° 119 Circle-Circle An equilateral triangle.

Remark : the option "clock" is not necessary because th line (BC) has a positive slope.



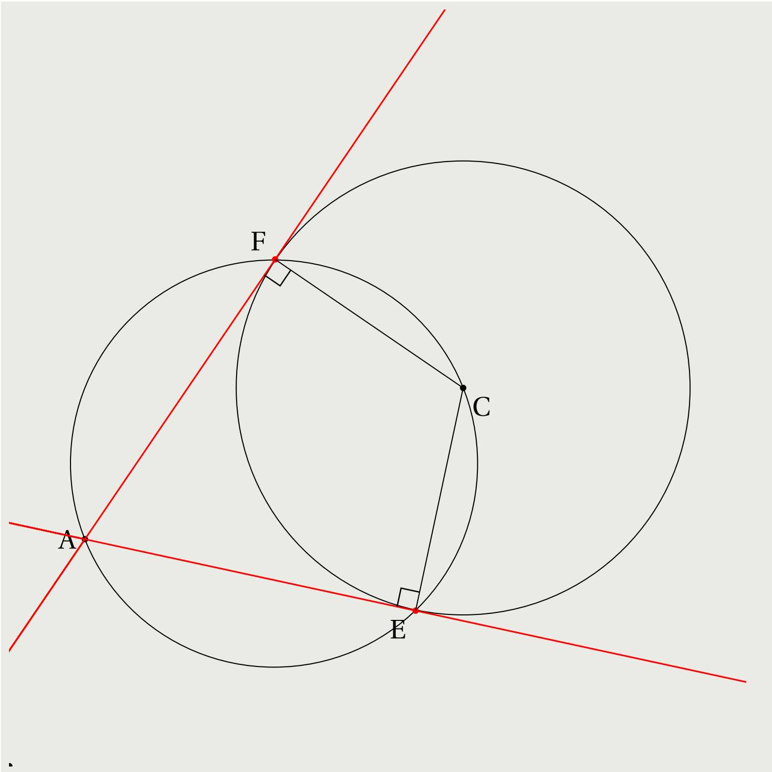

```
\begin{tikzpicture}
  \tkzInit[xmax=6,ymax=7]\tkzClip
  \tkzPoint[pos=left](1,4){A}
  \tkzPoint(5,1){B}
  \tkzSegment(A/B)
  \tkzInterCC(A,A,B)(B,B,A){C}{D}
  \tkzDrawPoint[pos={above},color=black](C)
  \tkzCompass[color=red](A,C)
  \tkzCompass[color=red](B,C)
  \tkzSegment(A/C,B/C)
\end{tikzpicture}
```

Example n° 120 Circle-Circle Mediator Line.




```
\begin{tikzpicture}
  \tkzInit[xmax=7,ymax=7]
  \tkzClip
  \tkzPoint[pos=left](1,4){A}
  \tkzPoint(5,1){B}
  \tkzSegment(A/B)
  \tkzMidPoint[color=red,%
              pos=below](A,B){I}
  \tkzInterCCR(A,4 cm)(B,4 cm){C}{D}
  \tkzDrawPoint[pos    = right,%
                color = red,%
                size  = 1.5pt](C)
  \tkzDrawPoint[pos    = left,%
                color = red,%
                size  = 1.5pt](D)
  \tkzCompass[color=red](A,C)
  \tkzCompass[clock,color=red](B,C)
  \tkzCompass[color=red](A,D)
  \tkzCompass[clock,color=red](B,D)
  \tkzLine[color=red](C/D)
  \tkzRightAngle(A/I/C)
  \tkzSegmentMark[symbol=/](A/I,B/I)
\end{tikzpicture}
```

Example n° 121 Circle-Circle Tangent to a circle from a given point not on the circle



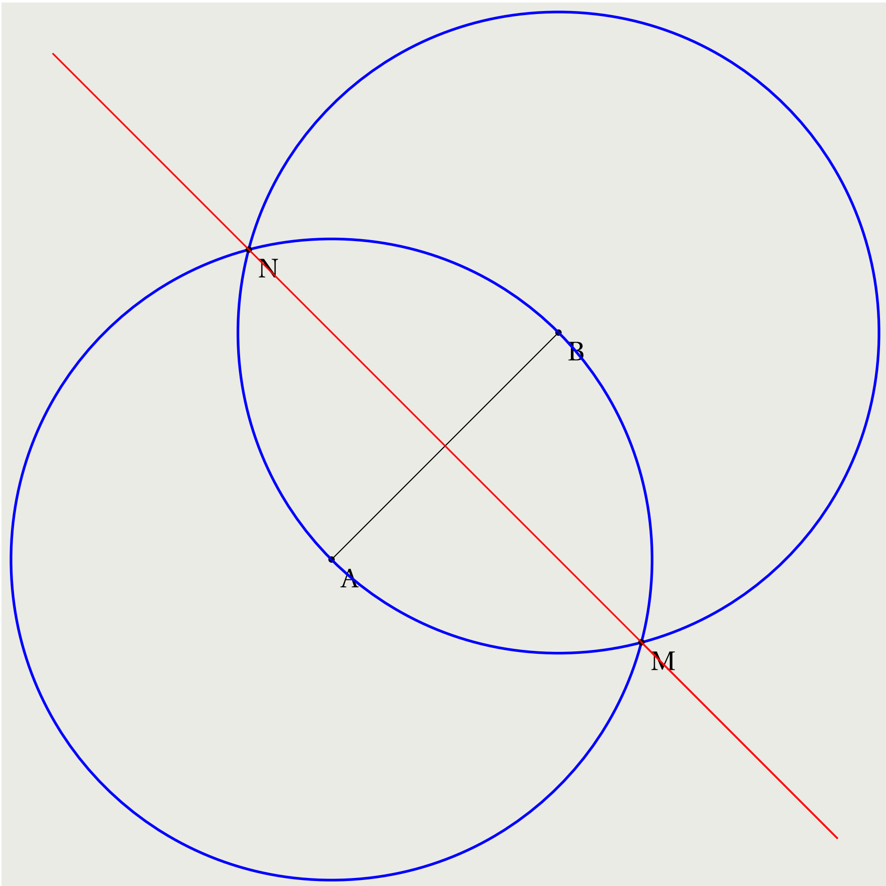

```
\begin{tikzpicture}
   \tkzrayon=3 cm\relax%
   \tkzInit\tkzClip
   \tkzPoint[pos=left](1,3){A}
   \tkzPoint(0,0){O}
   \tkzPoint(6,5){C}
   \tkzCircleR(C,\tkzrayon)
   \tkzMidPoint*(A,C){H}%
   \tkzCircleR*(H,H,C)
   \tkzInterCCR(C,\tkzrayon)(H,\tkzmathLen pt){E}{F}%
   \tkzDrawPoint[pos=above left,color=red](F)
   \tkzDrawPoint[pos=below left,color=red](E)
   \tkzLine[color=red](A/E,A/F)
   \tkzRightAngle(A/E/C,A/F/C)
   \tkzSegment(C/E,C/F)
\end{tikzpicture}
```

macro n° 62 Intersection Circle-Circle \tkzInterLC

\tkzInterLC[⟨*Point options*⟩](⟨A, E, F⟩)(⟨B, G, H⟩){⟨M⟩}{⟨N⟩}

Determine the intersection (M *and* N*) of two circles with respectively, centers* A *and* B*, radii given by* EF *and* GH .

Example n° 122 Circle-Circle Mediator line of a segment




```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint(0,0){A} \tkzPoint(3,3){B}
  \tkzCircleR*[color=blue,lw=1pt](B,B,A)
  \tkzCircleR*[color=blue,lw=1pt](A,A,B)
  \tkzInterCC(B,B,A)(A,A,B){M}{N}
  \tkzSegment(A/B)
  \tkzDrawPoint(M,N)
  \tkzLine[color=red,kr=.5,kl=.5](M/N)
\end{tikzpicture}
```

IX . Geometric Transformations

Transformations

All the classical transformations are available and share the same syntax. The last argument is a list. One element of this list is a antecedent point and its image, separed by "/". In the case no image name is given, a default name is used, appending a ' to the antecedent point name.

`\Name[⟨local options⟩](⟨arg.⟩)(⟨`$M_1/M'_1,\dots,M_n/M'_n$`⟩)` *or*

`\Name[⟨local options⟩](⟨arg.⟩)(⟨`$M_1/,\dots,M_n/$`⟩)`

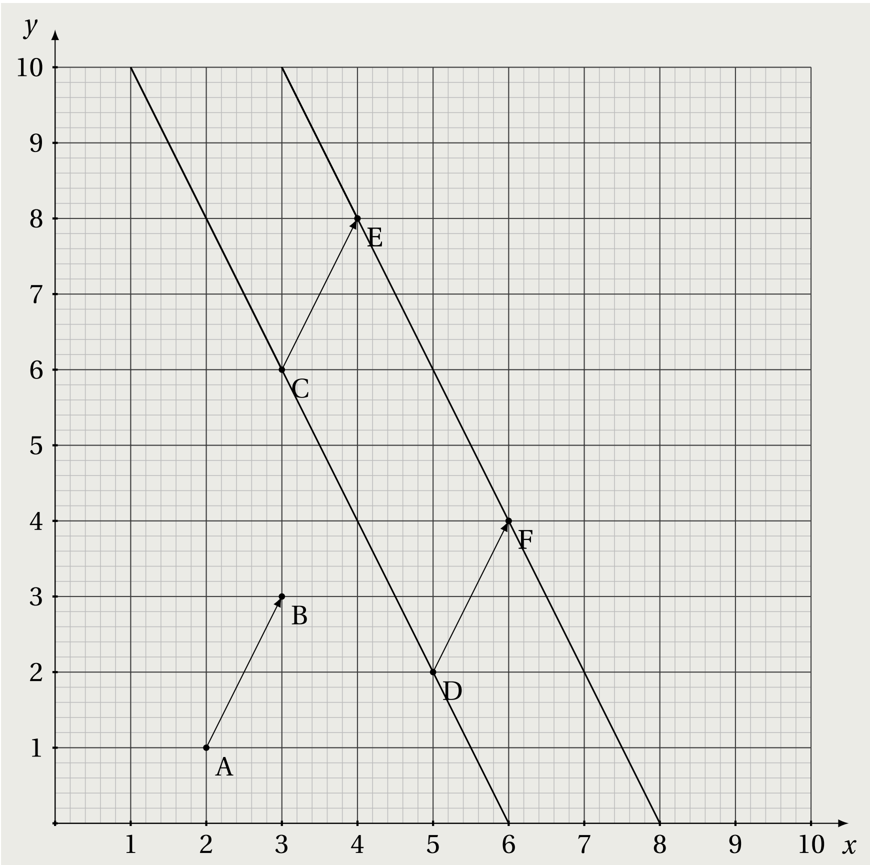
Each geometric transformation macro have a star form, when the star-form is used, the image point is not drawing. Local options are the same ones that for the point. If a star-form is used, you can not give local options.

macro n° 63 **Translation** `\tkzTranslation`

`\tkzTranslation[⟨local options⟩](⟨A,B⟩)(⟨`$M_1/M'_1,\dots,M_n/M'_n$`⟩)`

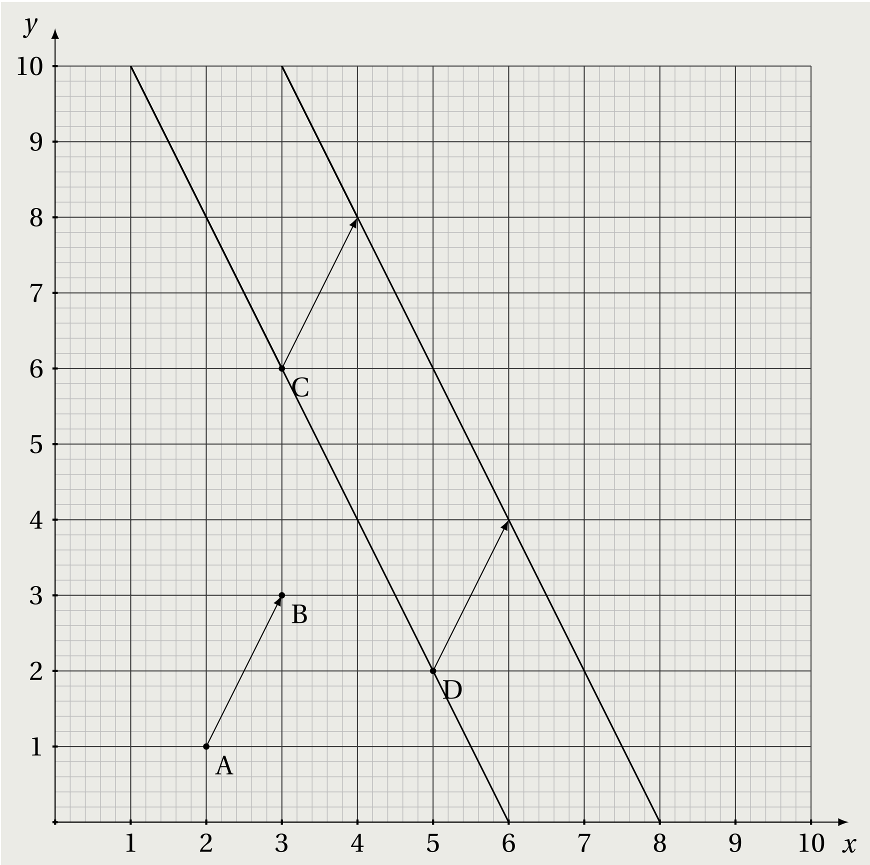
Translation is moving every point a constant distance in a specified direction. The direction is the direction of the vector defined by A *and* B *:* $\overrightarrow{AB}$ *and the distance is equal at* AB *(the length of* $\overrightarrow{AB}$*).* `\tkzTranslation(⟨A,B⟩)(⟨M/M'⟩)` *or* `\tkzTranslation(⟨A,B⟩)(⟨M/⟩)` *draws the translated point* M' *of point* M*, in other words* $\overrightarrow{AB} = \overrightarrow{MM'}$.

Example n° 123 Translation



```
\begin{tikzpicture}
  \tkzInit[] \tkzGrid[sub] \tkzX \tkzY \tkzClip
  \tkzPoints%
        (2/1/A,%
         3/3/B,%
         3/6/C,
         5/2/D)
    \tkzTranslation(A,B)(C/E,D/F)
    \tkzLine(C/D,E/F)
   \tkzDrawVector(A/B,C/E,D/F)
 \end{tikzpicture}
```

Example n° 124 Translation star form



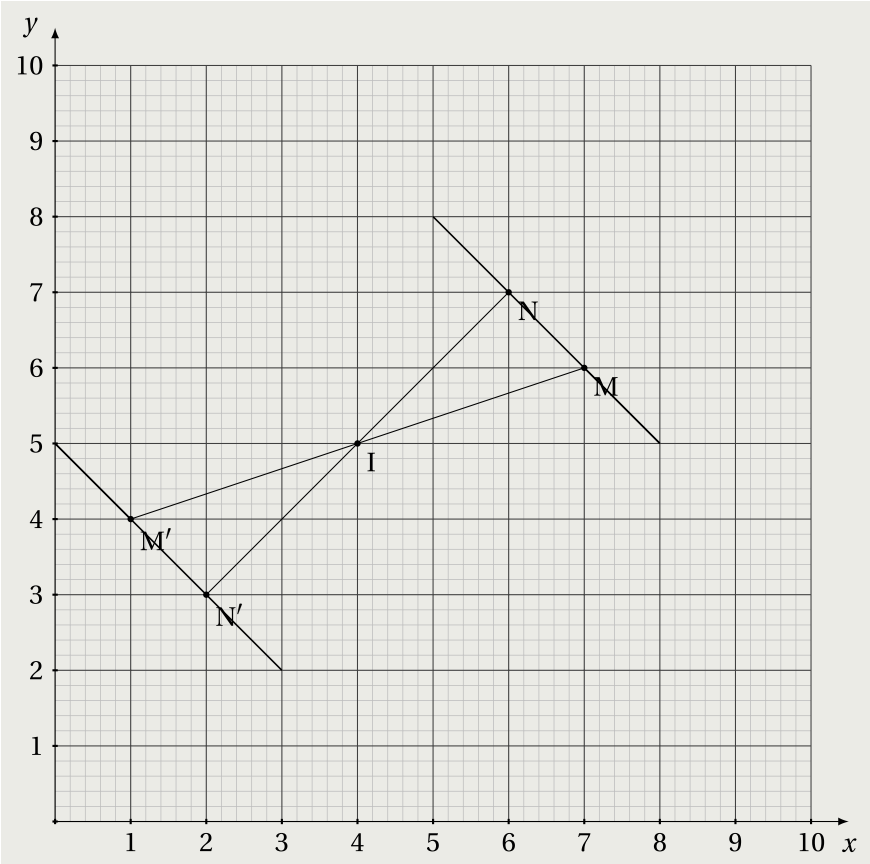

```
\begin{tikzpicture}
  \tkzInit[] \tkzGrid[sub] \tkzX \tkzY \tkzClip
 \tkzPoints%
        (2/1/A,%
         3/3/B,%
         3/6/C,
         5/2/D)
    \tkzTranslation*(A,B)(C/,D/)
    \tkzLine(C/D,C'/D')
   \tkzDrawVector(A/B,C/C',D/D')
 \end{tikzpicture}
```

macro n° 64 Central Symmetry \tkzCSym

\tkzCSym[⟨*local options*⟩](⟨I⟩)(⟨$M_1/M'_1,\ldots,M_n/M'_n$⟩)

Draw or define (star-form) the symmetryc point M' *of point* M *relatively a center* I.

Example n° 125 Central Symmetry



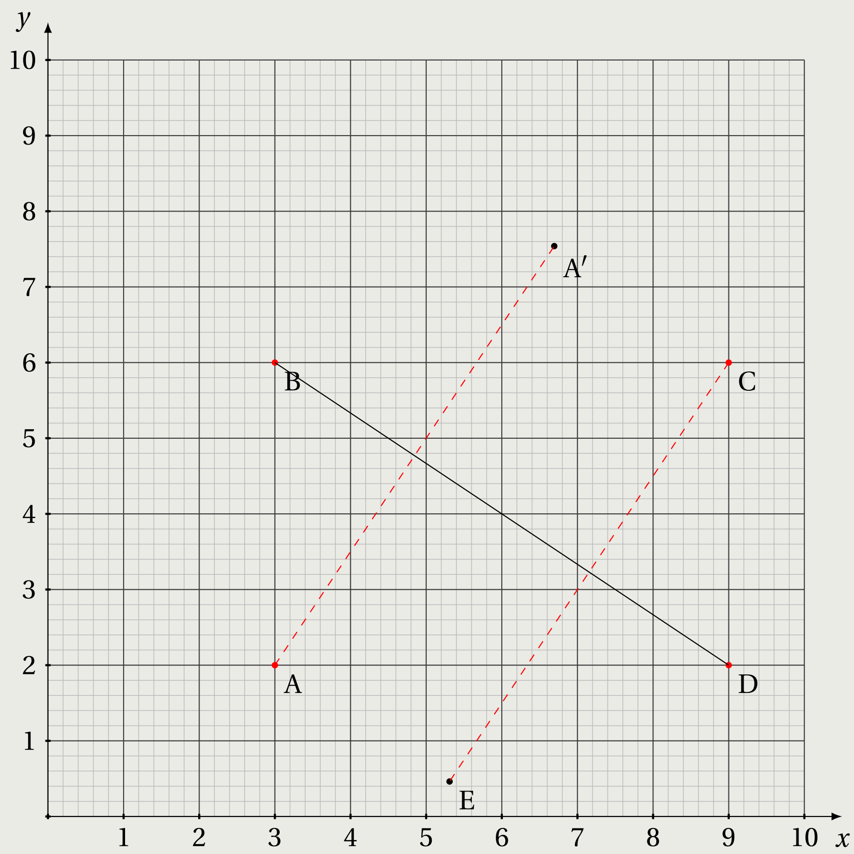

```
\begin{tikzpicture}
  \tkzInit \tkzGrid[sub] \tkzX \tkzY
  \tkzPoint(4,5){I}
  \tkzPoint(7,6){M}
  \tkzPoint(6,7){N}
  \tkzCSym(I)(M/,N/)
  \tkzSegment(M/M',N/N')
  \tkzLine(M/N,M'/N')
 \end{tikzpicture}
```

macro n° 65 Orthogonal Symmetry `\tkzSymOrtho`

`\tkzSymOrtho[⟨local options⟩](⟨A,B⟩)(⟨M1/M'1,...,Mn/M'n⟩)`

Draw or define (star-form) the symmetryc point M' *of point* M *in relation to line* (AB).

Example n° 126 Orthogonal Symmetry



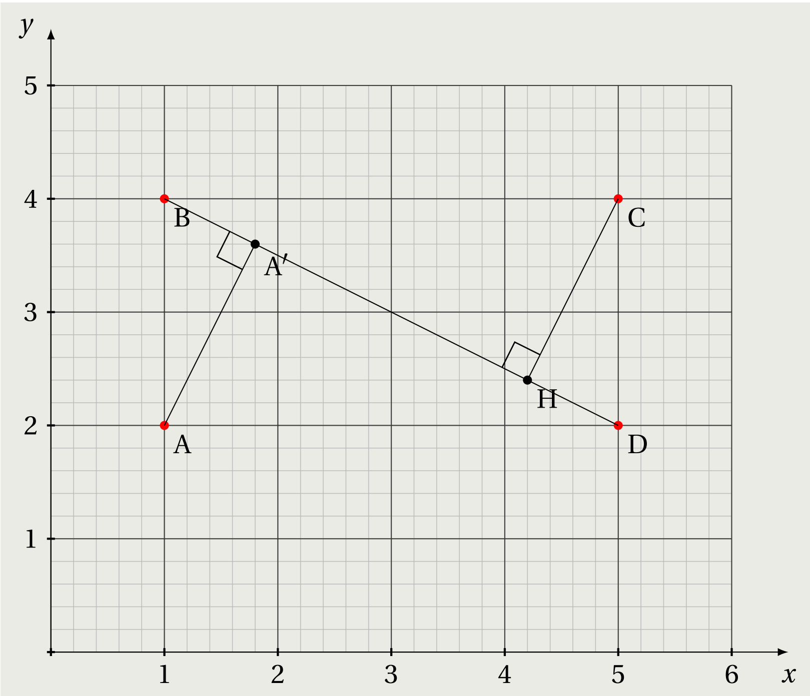

```
\begin{tikzpicture}
   \tkzInit
   \tkzGrid[sub]
   \tkzX
   \tkzY
   \path[coordinate]
         (3,2) coordinate (A)%
         (9,6) coordinate (C)%
               coordinate (B) at (A |- C)%
               coordinate (D) at (A -| C);
    \tkzDrawPoint[color=red](A)%
    \tkzDrawPoint[color=red](B)%
    \tkzDrawPoint[color=red](C)%
    \tkzDrawPoint[color=red](D)%
    \tkzSegment(B/D)
    \tkzSymOrth(B,D)(A/,C/E)
    \tkzSegment[style=dashed,color=red](A/A',C/E)
\end{tikzpicture}
```

macro n° 66 Orthogonal projection `\tkzProjection`

`\tkzProjection[⟨local options⟩](⟨A,B⟩)(⟨`$M_1/M'_1,\dots,M_n/M'_n$`⟩)`

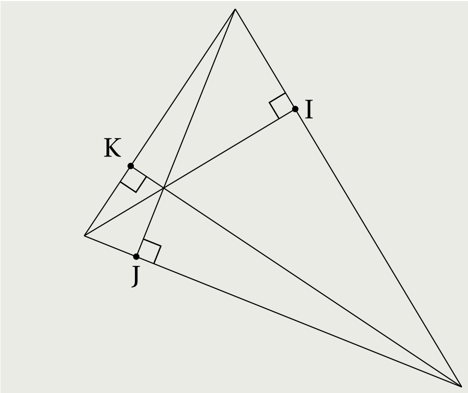
Draw or define (star-form) the projection point M' *of point* M *corresponding to the orthogonal projection onto the* (AB) *axis. Can be use to find a altitude in a triangle.*

Example n° 127 Orthogonal projection




```
\begin{tikzpicture}[scale=1.5]
  \tkzInit[xmax=6,ymax=5]
  \tkzGrid[sub]
  \tkzX
  \tkzY
  \tkzClip
  \path[coordinate]
       (1,2) coordinate (A)%
       (5,4) coordinate (C)%
             coordinate (B) at (A |- C)%
             coordinate (D) at (A -| C);
  \tkzDrawPoint[color=red](A)%
  \tkzDrawPoint[color=red](B)%
  \tkzDrawPoint[color=red](C)%
  \tkzDrawPoint[color=red](D)%
  \tkzProjection(B,D)(A/,C/H)
  \tkzSegment(A/A',C/H,B/D)
  \tkzRightAngle(B/A'/A,B/H/C)
\end{tikzpicture}
```

Example n° 128 Orthogonal projection in a triangle



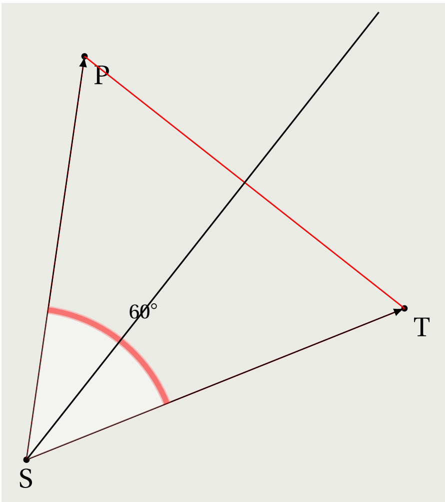

```
\begin{tikzpicture}[scale=1]
  \tkzInit[xmax=6,ymax=5] \tkzClip
  \path[coordinate]
       (1,2) coordinate (A)%
       (6,0) coordinate (C)%
       (3,5) coordinate (B);
  \tkzPolygon(A,B,C)%
  \tkzProjection[pos=above left](A,B)(C/K)
  \tkzProjection[pos=below](A,C)(B/J)
  \tkzProjection[pos=right](B,C)(A/I)
  \tkzSegment(A/I,B/J,C/K)
  \tkzRightAngle(A/I/B,B/J/C,C/K/A)
 \end{tikzpicture}
```

macro n° 67 Rotation en degré `\tkztRotate`

`\tkzRotate[⟨local options⟩](⟨I/angle⟩)(⟨M1/M'1,...,Mn/M'n⟩)`

Draw or define (star-form) the image M' *of point* M *by the rotation of center* I *and by a angle specified in degrees.*

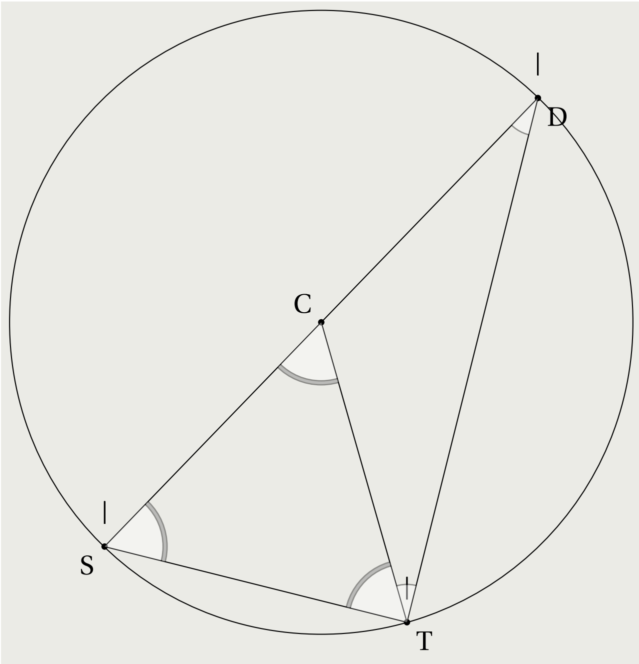
Example n° 129 Rotation 1




```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint[pos=below](2,1){S}
  \tkzPoint[](7,3){T}
  \tkzRotate(S,60)(T/P)
  \tkzPolygon[color=red,lw=0.5pt](S,T,P)
  \tkzDrawVector(S/P,S/T)
  \tkzMarkAngle[size  = 2,%
               style = aiii,%
               dist  = .5,%
               label = 60^{\circ},%
               color = red!50](T/S/P)%
  \tkzBisector[kr=.5,kl=0](T,S,P){z}
\end{tikzpicture}
```

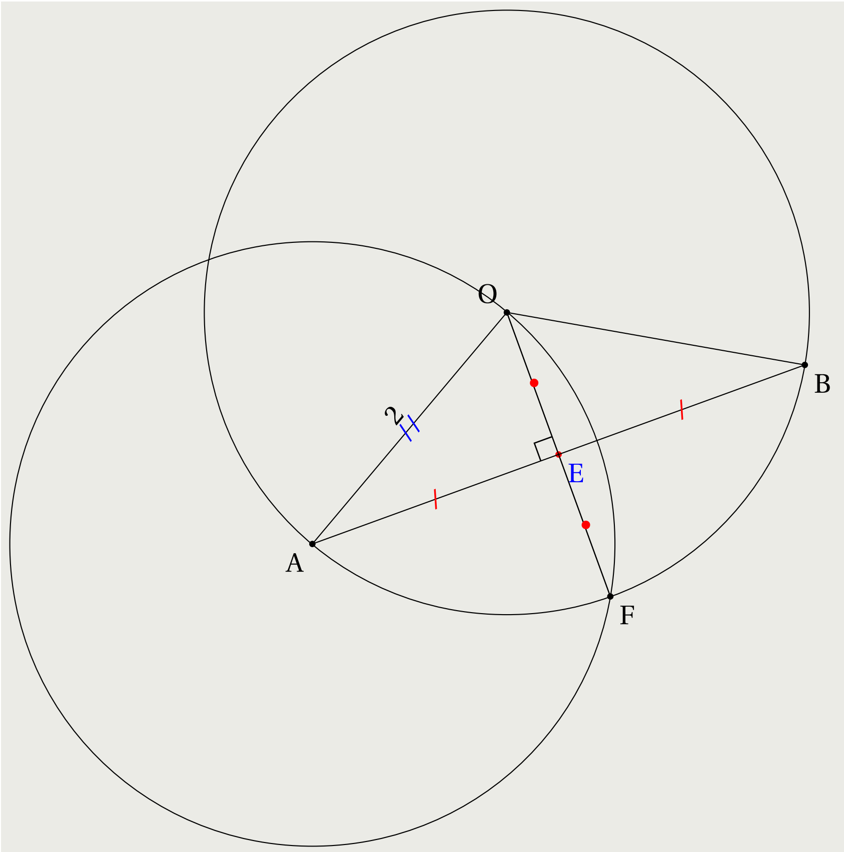
Example n° 130 Rotation 2

You can see some examples more complicated to obtain the measure in degrees of angles.




```
\begin{tikzpicture}
  \tkzInit
  \tkzPoint[pos=below left](2,2){S}
  \tkzPoint[pos=below right](6,1){T}
  \tkzRotate[pos={above left}](S,60)(T/C)
  \tkzRotate(C,120)(T/D)
  \tkzSegment(S/T,S/C,C/T,C/D,D/T)
  \tkzCircle(C,T)
  \tkzRightAngle[size=0.3](S,T,D)
  \tkzMarkAngle[size=.5](C/T/D)
  \tkzMarkAngle[size=.5](T/D/C)
  \tkzMarkAngle[style=aii,size=.8](C/S/T)
  \tkzMarkAngle[style=aii,size=.8](C/T/S)
  \tkzMarkAngle[style=aii,size=.8](S/C/T)
 \end{tikzpicture}
```

Example n° 131 Rotation 3



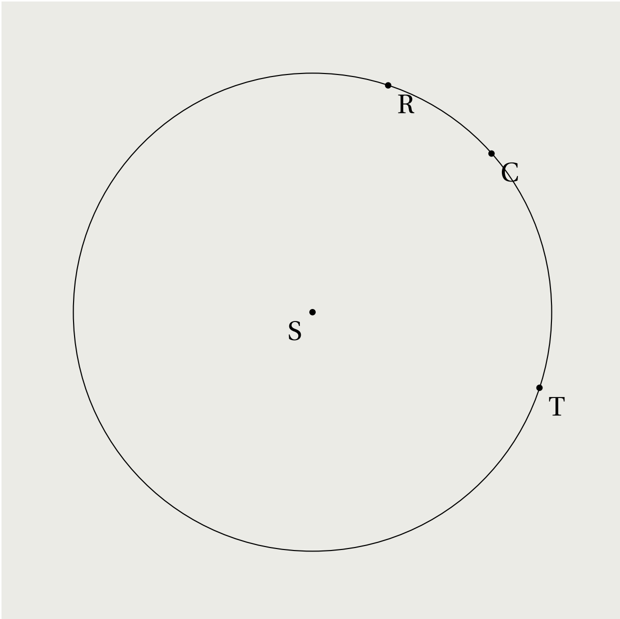

```
\begin{tikzpicture}[scale=1]
  \tkzInit[ymin=-5,ymax=5]
  \tkzPoint[pos=above left](0,0){O}
  \tkzCircleR(O,4cm)
  \tkzPoint*(0,-4){Temp}
  \tkzRotate[pos=below left](O,-40)(Temp/A)
  \tkzRotate(O,60)(A/F,F/B)
  \tkzSegment[](O/A,O/B)
  \tkzSegment(O/F)
  \tkzInterLL[pos=below right,color=red,namecolor=blue](A,B)(O,F){E}
  \tkzSegment[symbol=/,colorsymbol=red](E/B,A/E)
  \tkzSegment[symbol=$\bullet$,colorsymbol=red](E/F,O/E)
  \tkzRightAngle(A/E/O)
  \tkzSegmentMark[label=2,symbol=//,colorsymbol=blue](O/A)
  \tkzCircleR*(A,A,F)
\end{tikzpicture}
```

macro n° 68 Rotation en radian \tkzRotateR

\tkzRotateR[⟨*local options*⟩](⟨I/*angle in rad*⟩)(⟨$M_1/M'_1,\ldots,M_n/M'_n$⟩)

Draw or define (star-form) the image M' *of point* M *by the rotation of center* I *and by a angle specified in radians.*

Example n° 132 Rotation



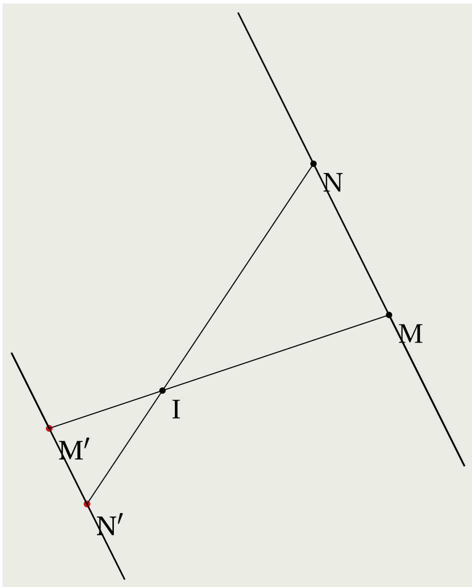

```
\begin{tikzpicture}
  \tkzInit[xmax=8,ymin=-2,ymax=6]
  \tkzClip
  \tkzPoint[pos=below left](4,2){S}
  \tkzPoint[pos=below right](7,1){T}
  \tkzRotateR(S,pi/3)(T/C)
  \tkzRotateR(S,pi/6)(C/R)
  \tkzCircle(S,R)
\end{tikzpicture}
```

macro n° 69 Homothethie \tkzHomo

\tkzHomo[⟨*local options*⟩](⟨I,*Ratio*⟩)(⟨$M_1/M'_1,\ldots,M_n/M'_n$⟩)

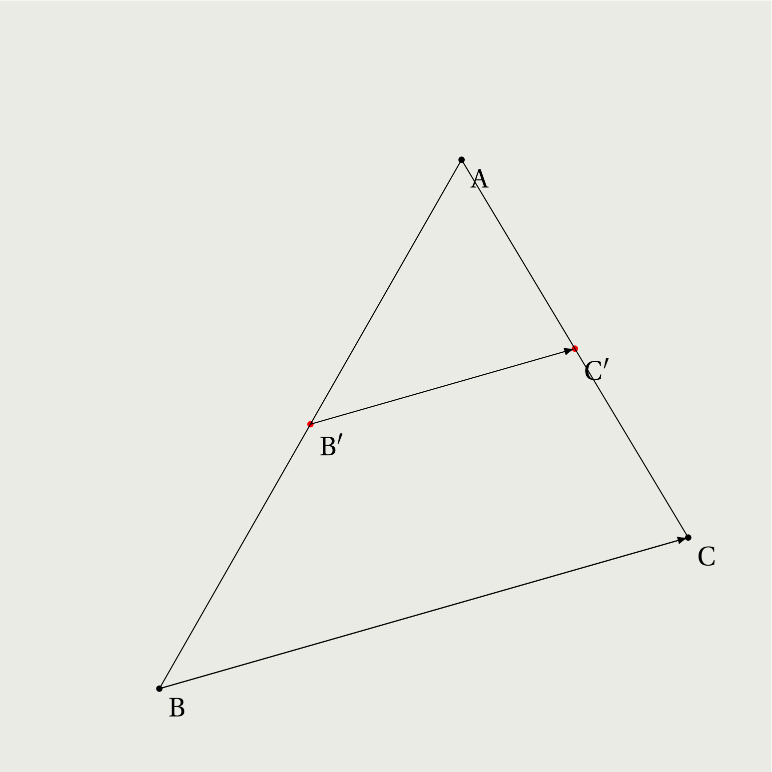
Homothety, dilation, central similarity are all interchangeable terms. Draw or define (star-form) the image M' *of point* M *by the homothety of center* I *and a decimal number "ratio".*

Example n° 133 Homothetie $-1 \leqslant$ ratio $\leqslant 0$




```
\begin{tikzpicture}
 \tkzInit
 \tkzPoint(4,5){I}
 \tkzPoint(7,6){M}
 \tkzPoint(6,8){N}
 \tkzHomo(I,-0.5)(M/,N/)
 \tkzDrawPoint[color=red](M')
 \tkzDrawPoint[color=red](N')
 \tkzLine(M/N,M'/N')
 \tkzSegment(M/M',N/N')
\end{tikzpicture}
```

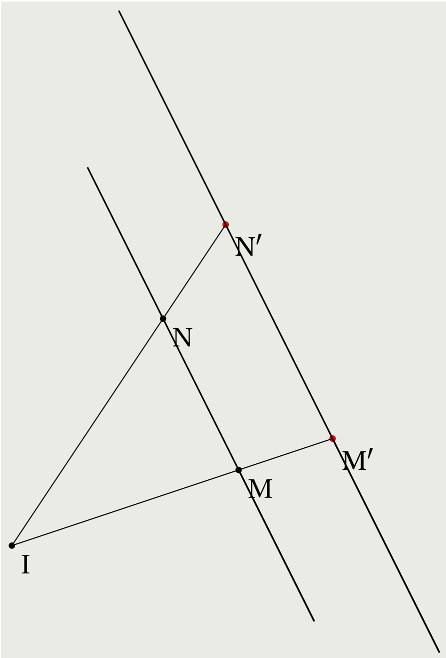
Example n° 134 Homothetie $0 \leqslant$ ratio $\leqslant 1$




```
\begin{tikzpicture}
 \tkzInit
 \tkzClip
 \tkzPoint(6,8){A}
 \tkzPoint(2,1){B}
 \tkzPoint(9,3){C}
 \tkzHomo[color=red](A,0.5)(B/,C/)
 \tkzPolygon(A,B,C)
 \tkzDrawVector(B/C,B'/C')
\end{tikzpicture}
```

Example n° 135 Homothetie ratio $=\sqrt{2}$

You can use all the functions defined in `pgfmath`.




```
\begin{tikzpicture}
 \tkzInit
 \tkzPoint(4,5){I}
 \tkzPoint(7,6){M}
 \tkzPoint(6,8){N}
 \tkzHomo(I,{sqrt(2)})(M/,N/)
 \tkzDrawPoint[color=red](M')
 \tkzDrawPoint[color=red](N')
 \tkzLine(M/N,M'/N')
 \tkzSegment(I/M',I/N')
\end{tikzpicture}
```

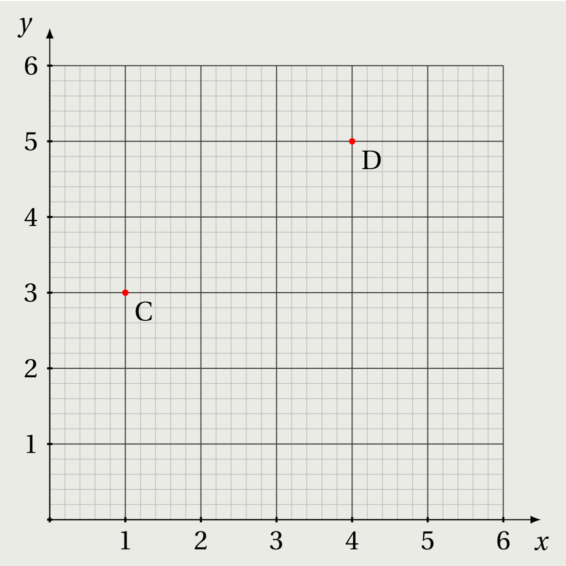
X. Tools

macro n° 70 Get coordinates of a node (point) \tkzGetPointxy

\tkzGetPointxy(⟨A⟩){⟨*text*⟩}

Define if text=p , two macros \px *and* \py*. These macros give the coordinates of the point* A *with cm for unit.* \px *and* \py *are numbers.*

Example n° 136 tkzGetPointxy



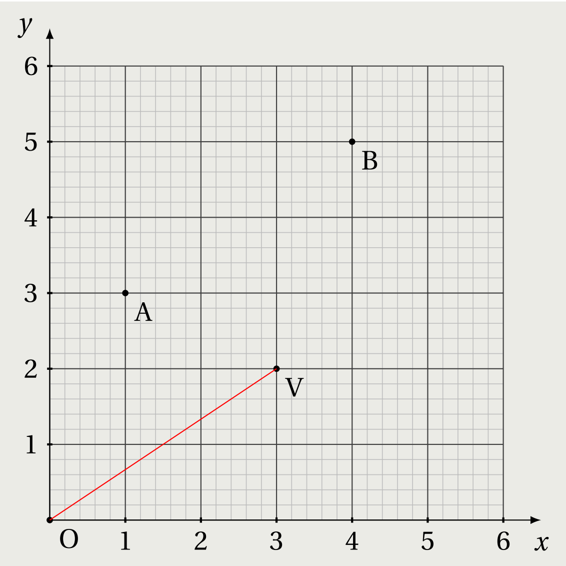

```
\begin{tikzpicture}
 \tkzInit[xmax=6,ymax=6]
 \tkzGrid[sub]
 \tkzX\tkzY
 \tkzPoint*(1,3){A}
 \tkzPoint*(4,5){B}
 \tkzGetPointxy(A){a}
 \tkzGetPointxy(B){b}
 \tkzPoint[color=red](\ax,\ay){C}
 \tkzPoint[color=red](\bx,\by){D}
\end{tikzpicture}
```

macro n° 71 Get coordinates of a vector \tkzGetVectxy

\tkzGetVectxy(⟨A,B⟩){⟨*text*⟩}

Define, if text=v, two macros \vx *and* \vy*. These macros give the coordinates of the vector* $\overrightarrow{\mathrm{AB}}$ *with cm for unit.*

Example n° 137 tkzGetVectxy



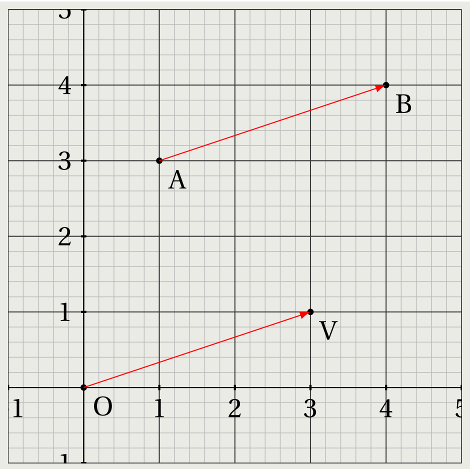

```
\begin{tikzpicture}
 \tkzInit[xmax=6,ymax=6]
 \tkzGrid[sub]
 \tkzX\tkzY
 \tkzPoint(0,0){O}
 \tkzPoint(1,3){A}
 \tkzPoint(4,5){B}
 \tkzGetVectxy(A,B){v}
 \tkzPoint(\vx,\vy){V}
 \draw[color=red](O)--(V);
\end{tikzpicture}
```

macro n° 72 Draw vector(s) \tkzDrawVector

\tkzDrawVector[⟨local options⟩](⟨A/B,C/D,...⟩)

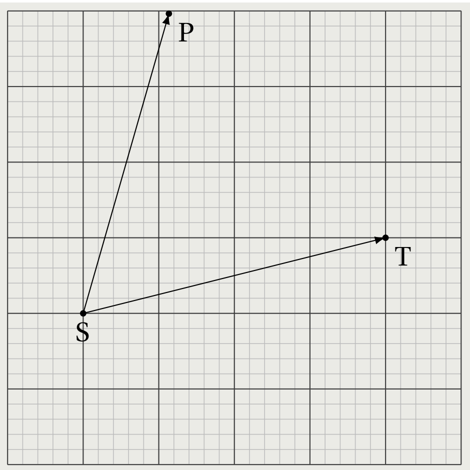
draw the vector $\overrightarrow{AB}$ *or the vectors of a list.*

Example n° 138 Drawing vectors




```
\begin{tikzpicture}
   \tkzInit[xmin=-1,ymin=-1,xmax=5,ymax=5]
   \tkzClip
   \tkzGrid[sub]
   \tkzX\tkzY
   \tkzPoint(0,0){O}
   \tkzPoint(1,3){A}
   \tkzPoint(4,4){B}
   \tkzGetVectxy(A,B){v}
   \tkzPoint(\vx,\vy){V}
   \tkzDrawVector[color=red](O/V,A/B)
\end{tikzpicture}
```

Example n° 139 Drawing vectors and rotate



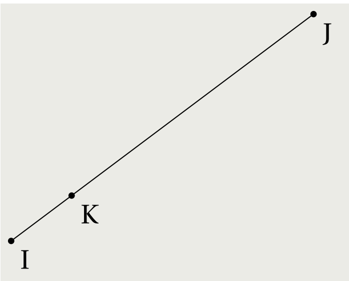

```
\begin{tikzpicture}
   \tkzInit[xmax=6,ymax=6]
   \tkzGrid[sub]
   \tkzPoint[pos=below](1,2){S}
   \tkzPoint(5,3){T}
   \tkzRotate(S,60)(T/P)
   \tkzDrawVector(S/P,S/T)
\end{tikzpicture}
```

macro n° 73 Get a point with a vector Normalised \tkzVectorNormalised

\tkzVectorNormalised(⟨A,B⟩){⟨C⟩}

We get a point C *such as*

Example n° 140 Vector Normalised




```
\begin{tikzpicture}[scale=1]
   \tkzInit[xmax=5,ymax=5]
   \tkzPoint(1,1){I}
   \tkzPoint(5,4){J}
   \tkzVectorNormalised(I,J){K}
   \tkzSegment(I/J)
   \tkzDrawPoint(K)
\end{tikzpicture}
```

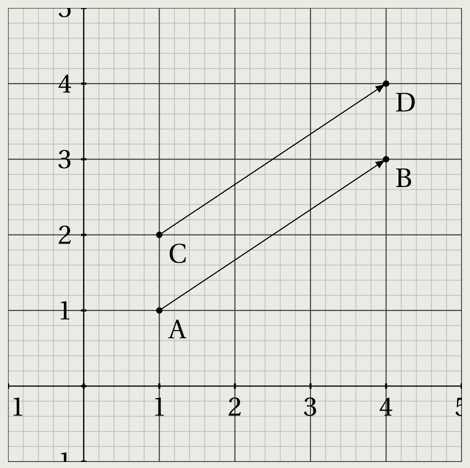
macro n° 74 Collinear Vector `\tkzVectorKLinear`

`\tkzVectorKLinear[⟨local options⟩](⟨A,B,C⟩){⟨D⟩}`

D *is point such as the line* (CD) *is a parallel line to* (AB) *such as* $\text{CD} = k \times \text{AB}$

Example n° 141 tkzVectorKLinear with $k = 1$

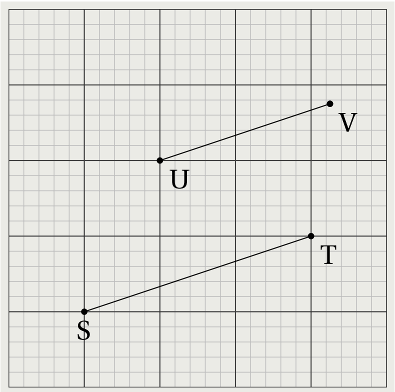
$\overrightarrow{\text{AB}} = \overrightarrow{\text{CD}}$




```
\begin{tikzpicture} [>=latex]
   \tkzInit[xmin=-1,ymin=-1,xmax=5,ymax=5]
   \tkzClip
   \tkzGrid[sub]
   \tkzX \tkzY
   \tkzPoint(1,1){A}
   \tkzPoint(4,3){B}
   \tkzPoint(1,2){C}
   \draw[->]  (A)--(B);
   \tkzVectorKLinear(A,B,C){D}
   \tkzDrawVector(C/D)
\end{tikzpicture}
```

Example n° 142 tkzVectorKLinear $k = .75$

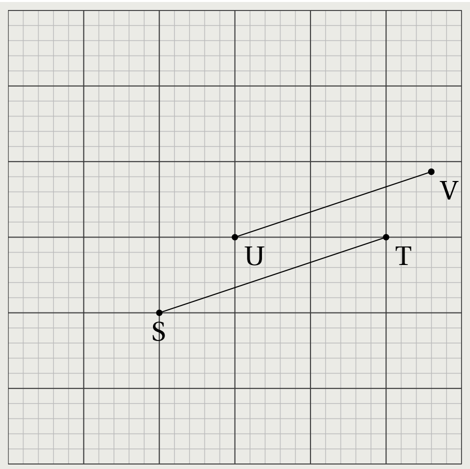
$\overrightarrow{\text{UV}} = \frac{3}{4}\overrightarrow{\text{ST}}$




```
\begin{tikzpicture}
 \tkzInit[xmax=5,ymax=5]
 \tkzClip
 \tkzGrid[sub]
 \tkzPoint[pos=below](1,1){S}
 \tkzPoint(4,2){T}
 \tkzPoint(2,3){U}
 \tkzVectorKLinear[k=.75](S,T,U){V}
 \tkzSegment(S/T,U/V)
 \tkzDrawPoint(V)
\end{tikzpicture}
```

Example n° 143 tkzVectorKLinear $k = \frac{\sqrt{3}}{2}$

$\overrightarrow{\text{UV}} = \frac{\sqrt{3}}{2}\overrightarrow{\text{ST}}$




```
\begin{tikzpicture}
 \tkzInit[xmin=-1,ymin=-1,xmax=5,ymax=5]
\tkzClip
 \tkzGrid[sub]
 \tkzPoint[pos=below](1,1){S}
 \tkzPoint[](4,2){T}
 \tkzPoint(2,2){U}
 \tkzVectorKLinear[k=sqrt(3)/2](S,T,U){V}
 \tkzSegment(S/T,U/V)
 \tkzDrawPoint(V)
\end{tikzpicture}
```

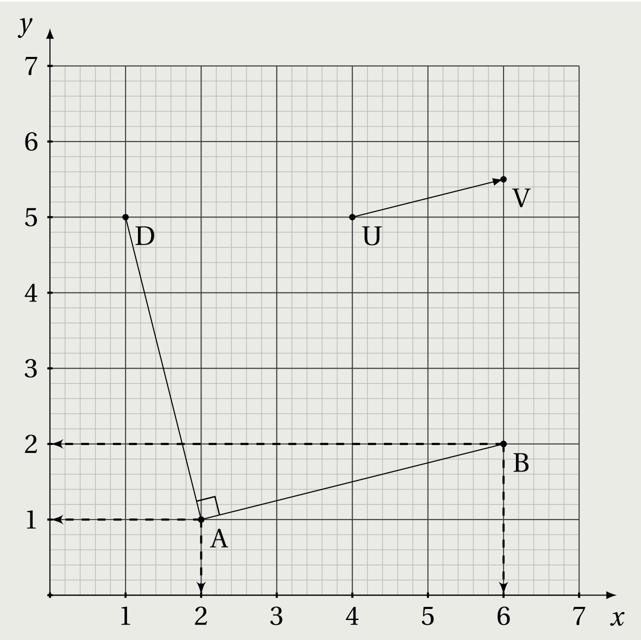
macro n° 75 Vector orthogonal `\tkzVectorOrth`

`\tkzVectorOrth[⟨local options⟩](⟨A,B⟩){⟨C⟩}`

We get a point C *such as* $\overrightarrow{AC}$ *is orthogonal to* $\overrightarrow{AC}$

Example n° 144 tkzVectorOrth $k = 0.5$

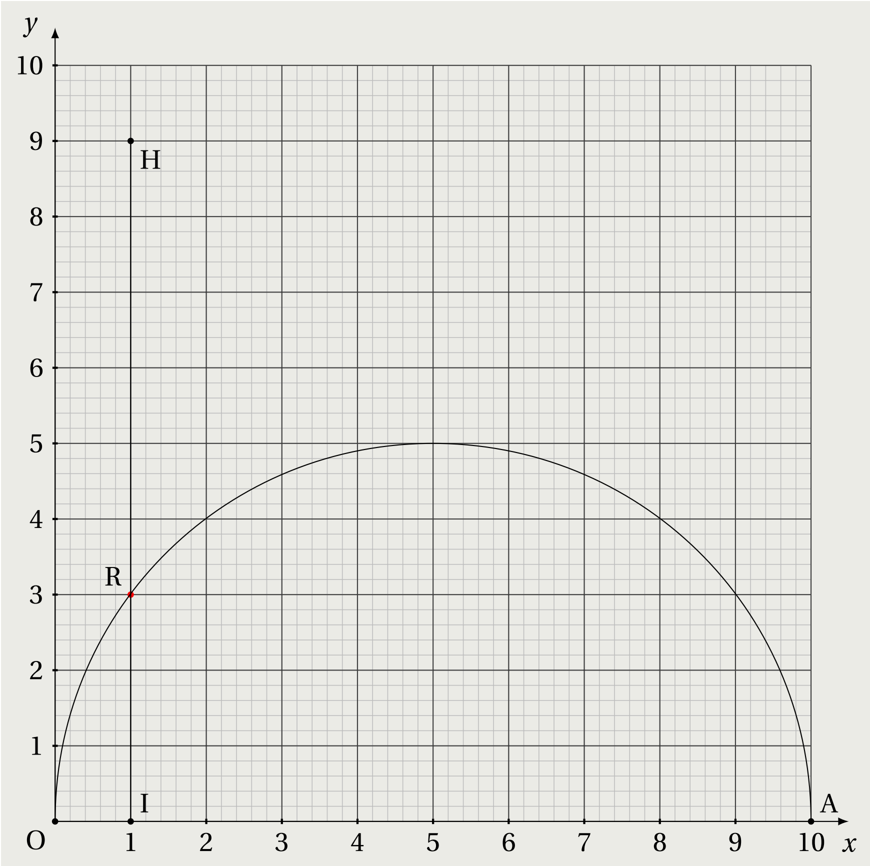
$\overrightarrow{AB}$ orthogonal to $\overrightarrow{AD}$ with AD = AB




```
\begin{tikzpicture}
   \tkzInit[xmax=7,ymax=7]
   \tkzGrid[sub]
   \tkzX\tkzY
   \tkzPoint[coord,pos={below right}](2,1){A}
   \tkzPoint[coord](6,2){B}
   \tkzPoint[](4,5){U}
   \tkzSegment(A/B)
   \tkzVectorOrth(A,B){D}
   \tkzSegment(A/D)
   \tkzRightAngle(B/A/D)
   \tkzVectorKLinear[k=0.5](A,B,U){V}
   \tkzDrawVector(U/V)
\end{tikzpicture}
```

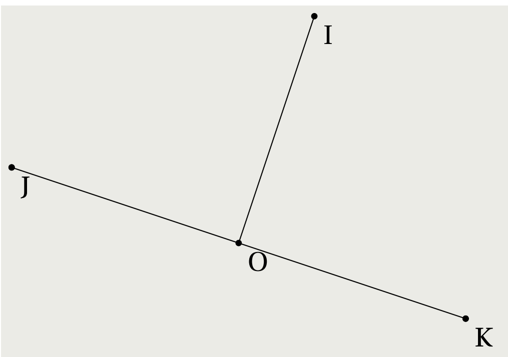
Example n° 145 tkzVectorOrth

How to get the square root of a number with a ruler and a compass.




```
\begin{tikzpicture}
   \tkzInit
   \tkzGrid[sub]
   \tkzX\tkzY
   \tkzPoint[pos=below left](0,0){O}
   \tkzPoint[pos=above right](1,0){I}
   \tkzPoint[pos=above left,color=red](1,3){R}
   \tkzPoint[pos=above right](10,0){A}
   \tkzVectorOrth(I,A){H}
   \tkzSegment(I/H)
   \tkzClip
   \tkzCircle*(O,A)
\end{tikzpicture}
```

Example n° 146 tkzVectorOrth $k = -1$



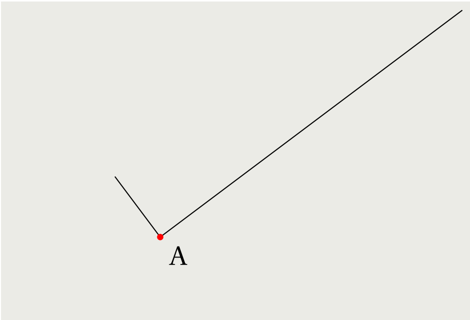

```
\begin{tikzpicture}
  \tkzInit[xmax=6,ymax=6]
  \tkzPoint(1,2){O}
  \tkzPoint(2,5){I}
  \tkzVectorOrth(O,I){J}
  \tkzVectorOrth[k=-1](O,I){K}
  \tkzDrawPoint(J,K)
  \tkzSegment(O/I,O/J,O/K)
\end{tikzpicture}
```

macro n° 76 Vector orthogonal and normalised `\tkzVectorOrthNormalised`

`\tkzVectorOrthNormalised(⟨A,B⟩){⟨C⟩}`

We get a point C *such as* $\overrightarrow{AC}$ *is orthogonal to* $\overrightarrow{AC}$

Example n° 147 Vector orthogonal and normalised $k = 0.5$



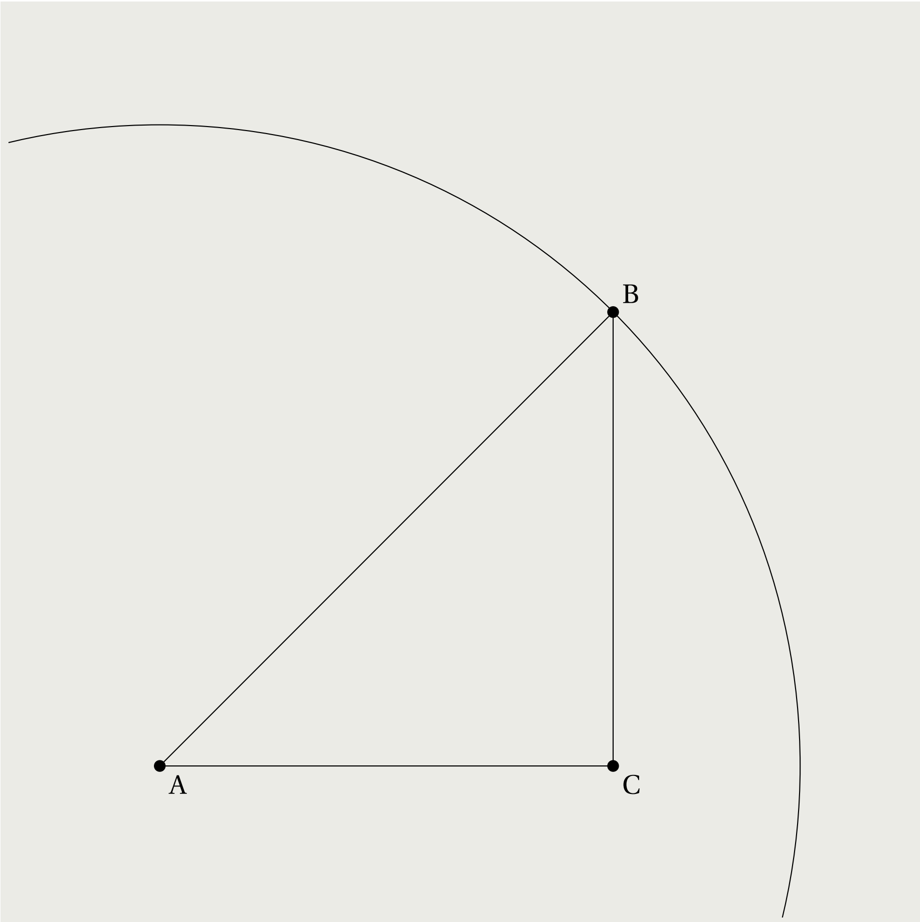

```
\begin{tikzpicture}
  \tkzInit
  \path[coordinate] (2,1) coordinate(A)%
                    (6,4) coordinate(B)
                    (0,0) coordinate(O);
   \tkzVectorOrthNormalised(A,B){C}
   \tkzSegment(A/B,A/C)
   \tkzDrawPoint[color=red](A)
\end{tikzpicture}
```

macro n° 77 Get a length `\tkzMathLength`

`\tkzMathLength(⟨A,B⟩)`

Find the measure of the length of segment AB *with unit in pt. The result is given in* `\tkzmathLen`.
AB = `\tkzmathLen` *pt.*

Example n° 148 length of segment AB



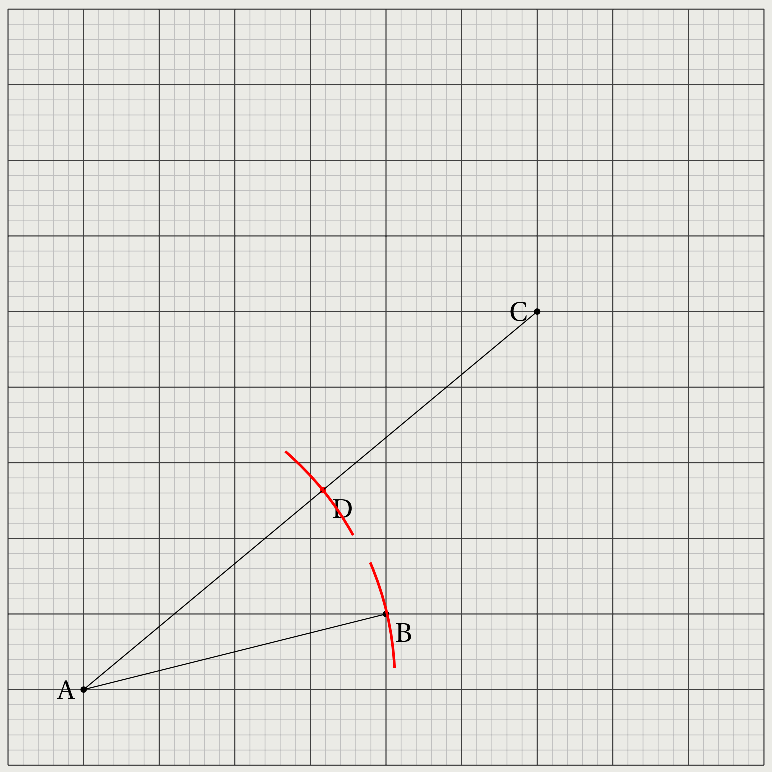

```
\begin{tikzpicture}[scale=2]
   \tkzInit[xmax=6,ymax=6] \tkzClip
   \tkzPoint(1,1){A}
   \tkzPoint[pos=above right](4,4){B}
   \tkzPoint(4,1){C}
   \tkzSegment(A/B,A/C,B/C)
   \tkzMathLength(A,B)
   \draw (A) circle(\tkzmathLen pt);
\end{tikzpicture}
```

macro n° 78 Duplicate a length `\tkzDuplicateLength`

`\tkzDuplicateLength(⟨A,B⟩)(⟨C,D⟩){⟨E⟩}`

Find a point E *on the line* (CD) *such as* CE = AB.

Example n° 149 Duplicate a segment



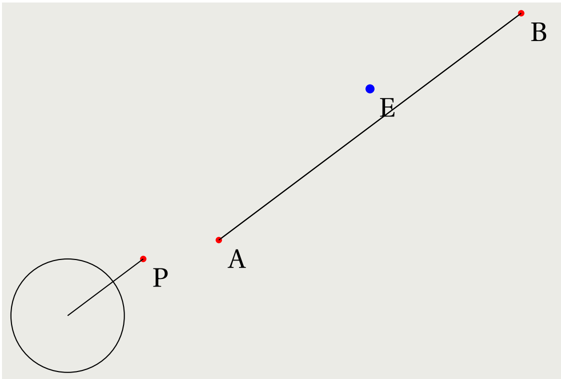

```
\begin{tikzpicture}
   \tkzInit\tkzGrid[sub]
   \tkzPoint[pos=left](1,1){A} \tkzPoint(5,2){B}
   \tkzPoint[pos=left](7,6){C}
   \tkzDuplicateLength[color=red](A,B)(A,C){D}
   \tkzSegment(A/B,A/C)
   \tkzCompass[color=red,lw=1pt](A,B)
   \tkzCompass[color=red,lw=1pt](A,D)
\end{tikzpicture}
```

macro n° 79 Find the slope of a line (AB) \tkzFindSlope

\tkzFindSlope(⟨A,B⟩)

Find the slope of the line (AB). *The result is a number given by* tkzSlope

Example n° 150 Duplicate a segment




```
\begin{tikzpicture}
 \tkzInit
 \path[coordinate] (2,1) coordinate(A)%
                   (6,4) coordinate(B)
                   (0,0) coordinate(O);
 \tkzSegment(A/B)
 \tkzDrawPoint[color=red](A)
 \tkzDrawPoint[color=red](B)
 \tkzFindSlope(A,B)
 \pgfmathparse{atan(\tkzSlope)}%
 \edef\tkzAlphaInD{\pgfmathresult}%
 \tkzPoint[color=red](1,\tkzSlope){P}
 \tkzCircleR(O,\tkzSlope cm)
 \tkzSegment(A/B,O/P)
 \pgfmathparse{cos(\tkzAlphaInD)}\global\let\px\pgfmathresult
 \pgfmathparse{sin(\tkzAlphaInD)}\global\let\py\pgfmathresult%
 \tkzPoint[size=1.5pt,color=blue](5*\px,5*\py){E}
\end{tikzpicture}
```